

# Exhibit “A”

**IN THE UNITED STATES DISTRICT COURT  
FOR THE EASTERN DISTRICT OF TEXAS**

**TYLER DIVISION**

BLUE SPIKE, LLC

*Plaintiff,*

v.

TEXAS INSTRUMENTS, INC., et al.

§ Civil Action No. 6:12-CV-499 MHS

§

§

§ LEAD CASE

§

§

§

---

**DEFENDANTS' PRELIMINARY CLAIM CONSTRUCTIONS**

Pursuant to Local Patent Rule 4-2, Defendants Iritech, Inc., NEUROtechnology, Fulcrum Biometrics, LLC, Futronic Technology Co., Ltd., iPharro Media GmbH, iPharro Media, Inc., Audible Magic Corporation, Photobucket.com, Inc., Qlipso, Inc., Qlipso Media Networks Ltd., Zedge Holdings, Inc., Mediafire, LLC., Dailymotion, Inc., Dailymotion S.A., GoMiso, Inc, iMesh, Inc., Coincident.TV, Inc., Facebook, Inc., MySpace, LLC, Specific Media, LLC, Yap.tv, Inc., Metacafe, Inc., Boodabee Technologies Inc., Harmonix Music Systems, Inc., Brightcove, Inc., Accedo Broadband AB, Accedo Broadband NA, Inc., Soundcloud, Inc., Soundcloud Ltd., WiOffer, LLC, Myxer, Inc., Attributor Corporation, Viggie, Inc., Shazam Entertainment Limited, MorphoTrust USA, Inc., L-1 Identity Solutions, Inc., MorphoTrak, Inc., Safran USA, Inc., Irdeto USA, Inc., Irdeto B.V., CBS Corp., Last.fm Ltd., CBS Interactive, Inc., Infinisource, Inc., Qquest Software Systems, Inc., SMRTV, Inc., The Nielsen Company (US) LLC, Clear Channel Broadcasting, Inc., 3M Cogent, Inc., Entropic Communications, Inc., Cognitec Systems GmbH, Cognitec Systems Corp., Civolution USA, Inc., Civolution B.V., and Airborne Biometrics Group, Inc., (collectively, "Defendants") hereby provide their preliminary constructions of claim

terms, phrases, or clauses of U.S. Patent Nos. 7,346,472 (the “472 patent”), 7,660,700 (the “700 patent”), 7,949,494 (the “494 patent”), and 8,214,175 (the “175 patent”).

Defendants submit these Preliminary Constructions for the purposes of limiting the terms in dispute and narrowing the issues as part of the meet-and-confer process outlined in Patent Local Rules 4-1(b) and 4-2(c); these constructions are preliminary and should not be considered to be admissions in any way. Defendants reserve the right to supplement their Preliminary Constructions to offer a construction, and/or to identify intrinsic and extrinsic evidence, for any term or phrase for which Blue Spike proffers a claim construction and for which Defendants do not offer a construction herein. Defendants also reserve the right to offer a construction for any subset of a term or phrase, or to construe a term or phrase in the context of additional claim language, as may be appropriate under the circumstances.

Copies of certain cited extrinsic evidence are attached hereto as Exhibit A. To the extent the above-identified extrinsic evidence has not been previously produced, Defendants will include such extrinsic evidence in a future document production, to the extent required to support any claim construction on which the parties do not agree.

Defendants’ discovery and investigation of Blue Spike’s claims is ongoing, and these Preliminary Constructions are based on information reasonably available to Defendants as of this date. Accordingly, Defendants reserve the right to amend, modify, and/or supplement this disclosure following: (1) any modification to Blue Spike’s Infringement Contentions; (2) the parties’ meet and confer pursuant to Patent Local Rules 4-1(b) and 4-2(c); (3) disclosure of and/or modification to Blue Spike’s claim construction positions; (4) further fact and/or expert discovery; or (5) as otherwise appropriate.

One or more Defendants intend to rely upon the expert testimony of John Snell to opine regarding indefiniteness. One or more Defendants intend to rely upon the expert testimony of Professor Matthew Turk to opine regarding indefiniteness. In addition, one or more Defendants may rely upon one or more of the following experts to rebut any expert testimony offered by Blue Spike or to establish the state of the art at the time of the alleged invention: Professor Matthew Turk, John Snell, Dr. John Strawn, Roger Dannenberg, Dr. Craig M. Arndt, and Dr. Kevin W. Bowyer.

The inclusion of a claim term or phrase disclosed herein is not intended to, and does not, constitute an admission that the term or phrase is capable of construction and not indefinite under 35 U.S.C. § 112, lacking enablement or adequate written description, or otherwise invalidating. Defendants expressly reserve the right to contest the validity of any claim of any of Blue Spike's patents-in-suit, or any term thereof, on any ground. To the extent Defendants have proposed that a term is indefinite, Defendants will address that term in their Motion for Summary Judgment of Indefiniteness to be filed on September 9, 2014.

### **Preliminary Claim Constructions**

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
1	Abstract	<b>472</b> (3,4,8,11,12) <b>700</b> (1,6,7,10,11,40,49,50) <b>494</b> (1,5,11,17,20,21,29) <b>175</b> (8,11,12,13,15,16,17)	<b>All Defendants (except Morpho Defendants)<sup>1</sup></b> “A data-reduced representation of a signal that is the smallest amount of data that can represent and differentiate two signals for a given predefined signal set and that retains a perceptual relationship with the original signal”  <b>Morpho Defendants</b> Indefinite  To the extent the Court finds this term is definite,

<sup>1</sup> 3M Cogent contends that the term “abstract” is indefinite. To the extent the Court finds it definite, 3M Cogent agrees with Defendants' construction.

	Term	Patent (claims)	Proposed Construction
			Morpho proposes: “a reduction that preserves an aesthetic quality of the original signal”
	Digital reference signal abstract	<b>175</b> (8,11,12,13,15,16,17)	<p><b>All Defendants (except Morpho Defendants)</b>  “A data-reduced representation of a reference signal that is the smallest amount of data that can represent and differentiate two signals for a given predefined signal set and that retains a perceptual relationship with the original signal”</p> <p><b>Morpho Defendants</b>  Indefinite</p> <p>To the extent the Court finds this term is definite, Morpho proposes: “a reduction that preserves an aesthetic quality of the original digital reference signal”</p>
	Query signal abstract	<b>494</b> (29) <b>175</b> (11,12,13,15,16)	“Abstract of the Query Signal”
	Digital representation	<b>175</b> (8)	Plain and ordinary meaning.
	First digital reference signal abstract	<b>175</b> (13,15)	First digital signal Abstract
	Digitally signed abstract	<b>700</b> (11,50) <b>494</b> (21)	“data that results from performing a Digital Signature on an Abstract”
	Hashed abstract	<b>700</b> (11,50) <b>494</b> (21)	“data that results from performing a Hash on an Abstract”
2	Match/Matches/ Matched	<b>472</b> (3,4,8,11,12) <b>700</b> (1) <b>494</b> (1,29) <b>175</b> (12,13,15)	<p>A Match - “an indistinguishable copy”</p> <p>Matches - “is indistinguishable from ”</p> <p>Matched – “was indistinguishable from”</p> <p>Extrinsic Evidence:  “match (A) A condition in which the values of corresponding components of two or more data items are equal. See also: hit. (B) To compare two or more data items to determine whether their corresponding components are equal as in definition ‘A’.” The IEEE Standard Dictionary of Electrical and Electronics Terms (6th Ed. 1997).</p>
3	A comparing device that compares/ a comparing device....that	<b>472</b> (11) <b>494</b> (1,11)	<p>Means plus function.</p> <p>Function: comparing</p>

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
	compares/ a comparing device for comparing		Structure: no structure or algorithm disclosed.  To the extent the Court determines this term is not means-plus-function, Defendants propose this term is indefinite. To the extent the Court finds this term is not indefinite, Defendants propose: “A separate hardware component of the computerized system [that compares/for comparing/able to compare]”.
4	A device configured to determine if a query signal matches any one plurality of reference signals	<b>494</b> (29)	Means plus function.  Function: determine if a Query Signal matches any one plurality of reference signals  Structure: no structure or algorithm disclosed.  To the extent the Court determines this term is not means-plus-function, Defendants propose this term is indefinite. To the extent the Court finds this term is not indefinite, Defendants propose: “A separate hardware component of the computerized system configured to determine if a Query Signal Matches any one plurality of Reference Signals”.
5	Versions of [a/the/said/“that one of said plurality of”] reference signal[s]	<b>700</b> (1,6) <b>494</b> (1,11,29) <b>175</b> (8) <b>700</b> (40)	“multiple variations of a particular Reference Signal”
6	Selectable criteria	<b>472</b> (11)	“Rules available for selection, which create different abstracts for a particular reference signal”
7	signal	<b>472</b> (11) <b>700</b> (1)	“an electrical wave used to convey information”
8	Reference signal	<b>472</b> (3,4,8,11) <b>700</b> (1,6,7,8,10,40,49) <b>494</b> (1,4,5,11,17,18,20,29)	“An uncompressed Signal representing an entire work”
9	Query signal	<b>472</b> (3,4,8,11) <b>700</b> (1,10,40,49) <b>494</b> (1,11,20,29)	“An uncompressed Signal representing an entire work to be analyzed”
10	Reference database	<b>472</b> (3,8,11)	“A Database containing Abstracts for a predefined set of Reference Signals”
	database	<b>700</b> (40) <b>175</b> (8,11,12,13,15,17)	Plain and ordinary meaning
11	Index of relatedness	<b>472</b> (11)	Indefinite  To the extent the Court believes that this term is

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
			not indefinite, then Defendants propose:  “a rank of closeness between the Query Signal itself and the abstracts stored in the Reference Database”
12	Related to	<b>700</b> (40) <b>494</b> (11)	“Matches”
13	Similar to	<b>175</b> (8,11,17)	Indefinite  To the extent the Court believes that this term is not indefinite, then Defendants propose: “looks or sounds the same as”
14	Identifies	<b>472</b> (11)	”to provide a unique label for”
	Identifying	<b>700</b> (40) <b>494</b> (1,11)	”providing a unique label for”
	Recording the identify [sic] of the reference signal	<b>472</b> (8)	“storing the unique label that corresponds to only one Reference Signal”  Extrinsic Evidence: “recording ... The process of storing information on some storage medium for later retrieval.” The IEEE Standard Dictionary of Electrical and Electronics Terms (6th Ed. 1997).
	To be identified	<b>494</b> (1,11)	”to provide a unique label for”
15	Creating at least one counter corresponding to one of said at least one reference signal	<b>472</b> (3,8)	“creating an element used for counting, which corresponds to a particular Reference Signal”
16	First digital reference signal abstract match recorder	<b>175</b> (15)	“an element used for counting, which corresponds to a particular Abstract”
17	Incrementing the counter....when a match is found	<b>472</b> (3,8)	“increasing the value of the element used for counting when a Match is found”
18	A compare result	<b>175</b> (11)	“data that indicates whether a Match between two abstracts was found”  Extrinsic Evidence: “compare ... To check two items, such as words, files, or numeric values, so as to determine whether they are the same or different. In a program, the outcome of a compare operation often determines which of two or more actions is taken next.” Computer Dictionary (Microsoft Press, 3d Ed. 1997).
19	Generating a report	<b>472</b> (4)	“creating a serial output which can be subsequently processed to determine the total

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
			number of times various Signals have been detected”
20	Distributing at least one signal based on the comparison step	<b>700</b> (51)	<p>“delivering at least one Signal resulting from the comparison to multiple recipients”</p> <p>Extrinsic Evidence:  “signal distributing (telephone switching systems) Delivering of signals from a common control to other circuits.” The IEEE Standard Dictionary of Electrical and Electronics Terms (6th Ed. 1997).</p>
21	perceptual characteristics representative of parameters to differentiate between versions of the reference signal	<b>494</b> (11) <b>700</b> (40)	“Perceptual characteristics, which represent parameters, that distinguish multiple Versions of a single Reference Signal”
	signal characteristic parameters configured to differentiate between versions of said reference signal	<b>494</b> (1)	<p>“parameters that characterize a signal that distinguish between multiple Versions of the same Reference Signal”</p> <p>Extrinsic Evidence:  “parameter - variable factor or characteristic.” The Hutchinson Dictionary of Computing Multimedia and the Internet (3d ed. 1999).</p> <p>“signal parameter (programmable instrumentation) - That parameter of an electrical quantity whose values or sequence of values convey information.” The IEEE Standard Dictionary of Electrical and Electronics Terms (6th Ed. 1997).</p>
	“signal characteristic parameters configured to differentiate between a plurality of versions of the reference signal.”	<b>700</b> (1)	“parameters that characterize a Signal that distinguish between multiple Versions of the same Reference Signal”
	signal characteristic parameters configured to differentiate between other versions of that one of said plurality of reference signals	<b>494</b> (29)	“parameters that characterize a Signal that distinguish between multiple Versions of the same Reference Signal”
	signal characteristic parameters that differentiate between said plurality of	<b>175</b> (8)	“parameters that characterize a Signal that distinguish between multiple Versions of a single visual work and multimedia work”



	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
	different versions of said visual work and said multimedia work		
22	Data describing a portion of the characteristics of its associated reference signal	<b>700</b> (7) <b>494</b> (17)	Indefinite.  To the extent the Court believes that this term is not indefinite, then Defendants propose:  “Information describing less than all of the signal characteristic parameters used to create the Abstract for its associated Reference Signal”
23	Perceptible characteristic	<b>700</b> (8) <b>494</b> (5,18)	“characteristic capable of being perceived by a person”
24	Cognitive characteristic	<b>700</b> (8) <b>494</b> (18)	“characteristic capable of being understood by a person”
25	Subjective characteristic	<b>700</b> (8) <b>494</b> (18)	“characteristic capable of being perceived differently by different people”
26	Perceptual quality	<b>700</b> (8) <b>494</b> (18)	“quality capable of being perceived by a person”  Extrinsic Evidence: “perception [PHYSIO] Recognition in response to sensory stimuli; the act or process by which the memory of certain qualities of an object is associated with other qualities impressing the senses, thereby making possible recognition of the object.” McGraw-Hill Dictionary of Scientific and Technical Terms (5th Ed. 1994).
27	Recognizable characteristic	<b>700</b> (8) <b>494</b> (18)	“characteristic capable of being visually or aurally perceived by a person”
28	Cognitive feature	<b>494</b> (5)	“A feature that is capable of being understood by a person”
29	Cryptographic protocol	<b>700</b> (10,11,49,50) <b>494</b> (20,21)	“An agreed upon procedure for transforming data in order to secure it”  Extrinsic Evidence: “Cryptography – The discipline embodying principles, means, and methods for the transformation of data in order to hide its information content, prevent its undetected modification, and/or prevent its unauthorized use.” The IEEE Standard Dictionary of Electrical and Electronics Terms (6th Ed. 1997).
30	Hash	<b>700</b> (11,50) <b>494</b> (21)	“A mathematical transform that maps a bit string of arbitrary length to a fixed length bit string to achieve uniqueness”  Extrinsic Evidence: “Hash function - A function that maps a bit string of arbitrary length to a fixed length bit string.”

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
			Digital Signature Standard, Federal Information Processing Standards Publication, United States Department of Commerce, FIPS PUB 186-4.
31	Digital signature	<b>700</b> (11,50) <b>494</b> (21)	Indefinite.  To the extent the Court believes that this term is not indefinite, then Defendants propose: “the result of a cryptographic transformation of data that, when applied to an Abstract, provides a mechanism for verifying origin authentication, data integrity and signatory non-repudiation”
32	security controller that controls access to a secured area	<b>472</b> (12) <b>494</b> (20)	Indefinite  To the extent the Court believes that this term is not indefinite, then Defendants propose: “a hardware device that prevents unauthorized access”
33	Reduced in size	<b>175</b> (8,11,17)	“compressed”
34	Digital representation of one of a plurality of different versions of a visual work and a multimedia work	<b>175</b> (8)	Plain and ordinary meaning.
35	Programmed or structured to use an algorithm to generate said digital reference signal abstract from said digital reference signal  Programmed or structured to use said algorithm to generate said digital query signal abstract from said digital query signal	<b>175</b> (16)	Indefinite
36	Psycho-acoustic model	<b>175</b> (17)	“model that determines acoustic parameters that are humanly-perceptible”
37	Psycho-visual model	<b>175</b> (17)	“model that determines visual parameters that are humanly-perceptible”
38	Wherein the system applies a cryptographic protocol	<b>700</b> (10)	Indefinite
39	The system of claim ... further comprising storing the hashed	<b>700</b> (11) <b>494</b> (21)	Indefinite

	<b>Term</b>	<b>Patent (claims)</b>	<b>Proposed Construction</b>
	abstract ...		
40	The system of claim ... based on the comparison step	<b>494</b> (22)	Indefinite
41	derived from one of a cognitive feature or perceptible characteristic	<b>494</b> (5)	Plain and ordinary meaning – “based on”
42	analyzing	<b>472</b> (3,11) <b>700</b> (1,40) <b>494</b> (11)	“closely examining”
43	monitoring	<b>472</b> (3,8,11) <b>700</b> (1)	“inspecting on a regular or ongoing basis”
44	information corresponding to two versions	<b>700</b> (6)	“information corresponding to two Versions”

Dated: June 27, 2014

/s/ Christopher J. Higgins

I. Neel Chatterjee (admitted E.D. Texas)  
Gabriel M. Ramsey (admitted E.D. Texas)  
ORRICK, HERRINGTON & SUTCLIFFE, LLP  
1000 Marsh Road  
Menlo Park, CA 94025  
Telephone: (650) 614-7400  
Facsimile: (650) 614-7401  
gramsey@orrick.com  
nchatterjee@orrick.com

Alyssa M. Caridis (admitted E.D. Texas)  
ORRICK, HERRINGTON & SUTCLIFFE, LLP  
777 S. Figueroa St.  
Suite 3200  
Los Angeles, CA 90017  
Telephone: (213) 629-2020  
Facsimile: (213) 612-2499  
acaridis@orrick.com

Christopher J. Higgins (*pro hac vice*)  
ORRICK, HERRINGTON & SUTCLIFFE, LLP

1152 15<sup>th</sup> Street, N.W.  
Washington, DC 20005-1706  
Telephone: (202) 339-8400  
Facsimile: (202) 339-8500  
chiggins@orrick.com

Eric H. Findlay (Texas Bar No. 00789886)  
Walter W. Lackey, Jr. (Texas Bar No. 24050901)  
FINDLAY CRAFT, P.C.  
102 N. College Ave., Suite 900  
Tyler, TX 75702  
Telephone: (903) 534-1100  
Facsimile: (903) 534-1137  
efindlay@findlaycraft.com  
wlackey@findlaycraft.com

*Counsel for Defendants Audible Magic, Corp.,  
Myspace LLC, Specific Media LLC, Photobucket.com,  
Inc., DailyMotion, Inc., DailyMotion S.A.,  
SoundCloud, Inc., SoundCloud Ltd., Myxer, Inc.,  
Qlipso, Inc., Qlipso Media Networks, Ltd., Yap.tv,  
Inc., GoMiso, Inc., iMesh, Inc., Metacafe, Inc.,  
Boodabee Technologies, Inc., Zedge Holdings, Inc.,  
Brightcove Inc., Coincident.TV, Inc., Accedo  
Broadband North America, Inc., Accedo Broadband  
AB, MediaFire, LLC, WiOffer LLC, and Harmonix  
Music Systems, Inc.*

/s/ Joshua R. Furman

Josh A. Krevitt  
(NY Bar No. 2568228)  
Benjamin Hershkowitz  
(NY Bar No. 2600559)  
R. Scott Roe  
(NY Bar No. 4480224)  
Joshua Furman  
(NY Bar No. 4612255)  
Gibson, Dunn & Crutcher LLP  
200 Park Avenue  
New York, NY 10166  
Ph. 212.351.4000  
Fax: 212.351.6210  
jkrevitt@gibsondunn.com  
bhershkowitz@gibsondunn.com  
jfurman@gibsondunn.com  
sroe@gibsondunn.com

Michael E. Jones  
SBN: 10929400  
[mikejones@potterminton.com](mailto:mikejones@potterminton.com)  
Allen F. Gardner  
SBN: 24043679  
[allengardner@potterminton.com](mailto:allengardner@potterminton.com)  
Daniel A. Noteware, Jr.  
SBN: 24051123  
[dannynoteware@potterminton.com](mailto:dannynoteware@potterminton.com)  
Potter Minton, PC  
110 North College, Suite 500  
Tyler, Texas 75702  
Tel: 903-597-8311  
Fax: 903-593-0846

*Counsel for Defendant  
Shazam Entertainment Limited*

/s/ Daniel Johnson, Jr.  
C. Erik Hawes (TX Bar No. 24042543)  
Email: [ehawes@morganlewis.com](mailto:ehawes@morganlewis.com)  
Thomas R. Davis (TX Bar No. 24055384)  
Email: [tdavis@morganlewis.com](mailto:tdavis@morganlewis.com)  
MORGAN, LEWIS & BOCKIUS LLP  
1000 Louisiana Street, Suite 4000  
Houston, Texas 77002  
(713) 890-5000 Telephone  
(713) 890-5001 Facsimile

Daniel Johnson, Jr. (Admitted *Pro Hac Vice*)  
Email: [djohnson@morganlewis.com](mailto:djohnson@morganlewis.com)  
MORGAN, LEWIS & BOCKIUS LLP  
One Market, Spear Street Tower  
San Francisco, CA 94105  
(415) 442-1000 Telephone  
(415) 442-1001 Facsimile

*Attorneys for Defendants MorphoTrust USA, Inc.,  
L-1 Identity Solutions, Inc., MorphoTrak, Inc., and  
Safran USA, Inc.*

/s/ Andrew P. Valentine  
Andrew P. Valentine (*Pro Hac Vice*)  
DLA PIPER LLP (US)

2000 University Ave.  
East Palo Alto, CA 94303  
Telephone: 650.833.2000  
Facsimile: 650.833.2001  
andrew.valentine@dlapiper.com

/s/ John Guaragna  
401 Congress Avenue  
Suite 2500  
Austin, TX 78701-3799  
Telephone: 512.457.7125  
Facsimile: 512.721.2325  
john.guaragna@dlapiper.com

*Counsel for Defendants  
Irdeto USA, Inc. and Irdeto B.V.*

/s/ Edward R. Reines  
Edward R. Reines  
Cal. Bar No. 135960 – Admitted to E.D. Texas  
Lead Attorney  
edward.reines@weil.com  
Byron Beebe  
Cal. Bar No. 235179 – Admitted to E.D. Texas  
byron.beebe@weil.com  
Andrew L. Perito  
Cal. Bar No. 269995 – Admitted to E.D. Texas  
WEIL, GOTSHAL & MANGES LLP  
201 Redwood Shores Parkway  
Redwood Shores, CA 94065  
Telephone: (650) 802-3000  
Facsimile: (650) 802-3100

*Counsel for Defendants CBS Corp., Last.fm Ltd., and  
CBS Interactive, Inc.*

/s/ Kristin L. Cleveland  
Kristin L. Cleveland (OR State Bar No. 001318 –  
Admitted to E.D. Texas)  
kristin.cleveland@klarquist.com  
Kristen L. Reichenbach (OR State Bar No. 115858)  
Kristen.reichenbach@klarquist.com  
John D. Vandenberg (OR State Bar No. 893755)  
john.vandenberg@klarquist.com

121 SW Salmon Street, Suite 1600  
Portland, OR 97204  
Telephone: (503) 595-5300  
Facsimile: (503) 595-5301

Eric H. Findlay (Texas Bar No. 00789886)  
Walter W. Lackey, Jr. (Texas Bar No. 24050901)  
FINDLAY CRAFT, P.C.  
102 N. College Ave., Suite 900  
Tyler, TX 75702  
Telephone: (903) 534-1100  
Facsimile: (903) 534-1137  
efindlay@findlaycraft.com  
wlackey@findlaycraft.com

*Attorneys for Defendant Contributor Corporation*

/s/ Eric H. Findlay

Eric H. Findlay (Texas Bar No. 00789886)  
Walter W. Lackey, Jr. (Texas Bar No. 24050901)  
FINDLAY CRAFT, P.C.  
102 N. College Ave., Suite 900  
Tyler, TX 75702  
Telephone: (903) 534-1100  
Facsimile: (903) 534-1137  
efindlay@findlaycraft.com  
wlackey@findlaycraft.com

*Attorneys for Defendants Iritech, Inc.,  
NEUROtechnology, Fulcrum Biometrics, LLC,  
Futronic Technology Co., Ltd., iPharro Media  
GmbH, and iPharro Media, Inc*

/s/ Eric H. Findlay

Eric H. Findlay (Texas Bar No. 00789886)  
Walter W. Lackey, Jr. (Texas Bar No. 24050901)  
FINDLAY CRAFT, P.C.  
102 N. College Ave., Suite 900  
Tyler, TX 75702  
Telephone: (903) 534-1100  
Facsimile: (903) 534-1137  
efindlay@findlaycraft.com  
wlackey@findlaycraft.com

Jordan A. Sigale  
Christopher Swickhamer  
LOEB & LOEB LLP  
321 North Clark Street, Suite 2300  
Chicago, IL 60654  
Telephone: 312-464-3109  
Facsimile: 312-276-8375  
Email: jsigale@loeb.com  
Email: cswickhamer@loeb.com

*Attorneys for Defendant Viggie, Inc.*

/s/ Teresa M. Corbin  
Teresa M. Corbin  
Teresa M. Corbin, CA Bar No. 132360  
(Admitted E.D. Texas)  
Bryan A. Kohm, CA Bar No. 233276  
(Admitted E.D. Texas)  
David Lacy Kusters, CA Bar No. 241335  
(Admitted E.D. Texas)  
Fenwick & West LLP  
555 California Street, 12th Floor  
San Francisco, California 94104  
Telephone: (415) 874-2300  
Facsimile: (415) 281-1350

Darren E. Donnelly, CA Bar No. 194335  
(Admitted E.D. Texas)  
Fenwick & West LLP  
Silicon Valley Center  
801 California Street  
Mountain View, California 94041  
Telephone: (650) 955-8500  
Facsimile: (650) 983-5200

*Attorneys for Defendants  
Infinisource, Inc.; Qquest Software Systems, Inc.;  
SMRTV, Inc.; and The Nielsen Company (US) LLC*

/s/ Steven J. Corr  
Tharan Greg Lanier (admitted E.D. Texas)  
California Bar No. 138784  
Jones Day  
1755 Embarcadero Road  
Palo Alto, CA 94303



Telephone: (650) 739-3939  
Facsimile: (650) 739-3900  
tglanier@jonesday.com

Steven J. Corr (admitted E.D. Texas)  
California Bar No. 216243  
Jones Day  
555 S. Flower Street  
Los Angeles, CA 90278  
Telephone: (213) 243-2327  
Facsimile: (213) 243-2539  
sjcorr@jonesday.com

*Attorneys for Defendants The Nielsen Company (US)  
LLC*

/s/ Ryan K. Yagura  
Ryan K. Yagura - TX S.B. #240755933  
ryagura@omm.com - CA S.B. #197619  
Nicholas J. Whilt (*pro hac vice*)  
nwhilt@omm.com – CA S.B. #247738  
O'Melveny & Myers LLP  
400 South Hope Street  
Los Angeles, California 90071-2899  
Telephone: (213) 430-6000  
Facsimile: (213) 430-6407  
Sarah A. Pfeiffer (*pro hac vice*)  
spfeiffer@omm.com – CA S.B. #278205  
O'Melveny & Myers LLP  
610 Newport Center Drive  
Newport Beach, California 92660  
Telephone: (949) 823-6900  
Facsimile: (949) 823-6994

*Attorneys for Clear Channel Broadcasting, Inc.*

/s/ Robert F. Kramer  
Robert F. Kramer  
CA Bar No. 181706 (admitted E.D. Tex.)  
C. Gideon Korrell  
CA Bar No. 284890 (admitted E.D. Tex.)  
DENTONS US, LLP  
1530 Page Mill Road, Suite 200  
Palo Alto, CA 94304  
Tel: (650) 798-0300

Fax: (650) 798-0310  
robert.kramer@dentons.com  
gideon.korrell@dentons.com

*Counsel for Defendants 3M Cogent, Inc.*

/s/ Jo Dale Carothers  
Alan H. Blankenheimer  
ablankenheimer@cov.com  
Jo Dale Carothers  
jcarothers@cov.com  
COVINGTON & BURLING LLP  
9191 Towne Centre Drive, 6th Floor  
San Diego, CA 92122-1225  
Tel: 858-678-1800  
Fax: 858-678-1600

Michael E. Jones  
SBN: 10929400  
Potter Minton, PC  
110 N. College, Suite 500  
Tyler, Texas 75702  
Tel: 903-597-8311  
Fax: 903-593-0846  
mikejones@potterminton.com

*Attorneys for Defendant  
Entropic Communications, Inc.*

/s/ Dwayne K. Goetzel  
Eric B. Meyertons  
Texas State Bar No. 14004400  
Dwayne K. Goetzel  
Texas State Bar No. 08059500  
Ryan T. Beard  
Texas State Bar No. 24012264  
MEYERTONS, HOOD, KIVLIN,  
KOWERT & GOETZEL, P.C.  
1120 S. Capital of Texas Hwy.  
Building 2, Suite 300  
Austin, Texas 78746  
(512) 853-8800 (telephone)  
(512) 853-8801 (facsimile)

*Attorneys For Defendant*

*Cognitec Systems Gmbh And  
Cognitec Systems Corp.*

/s/ Michael A. Molano

Michael A. Molano (*pro hac vice*)  
Ward Johnson (*pro hac vice*)  
MAYER BROWN LLP  
3000 El Camino Real  
Palo Alto, CA 94306-2112  
Phone: (650) 331-2000  
Fax: (650) 331-2060  
Email: wjohnson@mayerbrown.com  
Email: mmolano@mayerbrown.com

Kyle E. Friesen  
Texas Bar No. 24061954  
MAYER BROWN LLP  
700 Louisiana Street, Suite 3400  
Houston, Texas 77002-2730  
Phone: (713) 238-2691  
Fax: (713) 238-4691  
Email: kfriesen@mayerbrown.com

*Counsel for Defendants  
Civolution USA, Inc. and Civolution B.V.*

/s/ Reid E. Dammann

Reid E. Dammann (CA Bar No. 249031)  
r.dammann@mpglaw.com  
MUSICK PEELER LLP  
One Wilshire Boulevard, Suite 2000  
Los Angeles, CA 90017  
Houston, TX 77010-1018  
Telephone: (213) 629-7611  
Facsimile: (213) 624-1376

*Counsel for Defendant Airborne Biometrics Group,  
Inc.*

/s/ Orion Armon

Heidi L. Keefe (*pro hac vice*)  
Mark R. Weinstein (*pro hac vice*)  
COOLEY LLP

Five Palo Alto Square  
3000 El Camino Real  
Palo Alto, CA 94306-2155  
Phone: (650) 843-5000  
Fax: (650) 857-0663  
hkeefe@cooley.com  
mweinstein@cooley.com

Orion Armon (*pro hac vice*)  
Sara J. Bradford (*pro hac vice*)  
Janna K. Fischer (*pro hac vice*)  
COOLEY LLP  
380 Interlocken Crescent, Suite 900  
Broomfield, CO 80021  
Tel: (720) 566-4000  
Fax: (720) 566-4099  
oarmon@cooley.com  
sbradford@cooley.com  
jfischer@cooley.com

Deron R. Dacus (Texas Bar No. 00790553)  
The Dacus Firm, P.C.  
821 ESE Loop 323, Suite 430  
Tyler, TX 75701  
Tel: (903) 705-1117  
Fax: (903) 705-1117  
ddacus@dacusfirm.com

*Counsel for Defendant Facebook, Inc.*

# Exhibit A

# **The IEEE Standard Dictionary of Electrical and Electronics Terms**

**Sixth Edition**

**Standards Coordinating Committee 10, Terms and Definitions  
Jane Radatz, Chair**

This standard is one of a number of information technology dictionaries being developed by standards organizations accredited by the American National Standards Institute. This dictionary was developed under the sponsorship of voluntary standards organizations, using a consensus-based process.

ISBN 1-55937-833-6



When the IEEE Standards Board approved this standard on 10 December 1996, it had the following membership.

**Donald C. Loughry, Chair**      **Richard J. Holleman, Vice Chair**  
**Andrew G. Salem, Secretary**

Gilles A. Baril  
 Clyde R. Camp  
 Joseph A. Cannatelli  
 Stephen L. Diamond  
 Harold E. Epstein  
 Donald C. Fleckenstein  
 Jay Forster\*  
 Donald N. Heirman  
 Ben C. Johnson

E. G. "Al" Kiener  
 Joseph L. Koepfinger\*  
 Stephen R. Lambert  
 Lawrence V. McCall  
 L. Bruce McClung  
 Marco W. Migliaro  
 Mary Lou Padgett  
 John W. Pope

Jose R. Ramos  
 Arthur K. Reilly  
 Ronald H. Reimer  
 Gary S. Robinson  
 Ingo Rüschi  
 John S. Ryan  
 Chee Kiow Tan  
 Leonard L. Tripp  
 Howard L. Wolfman

\*Member Emeritus

Also included are the following nonvoting IEEE Standards Board liaisons:

Satish K. Aggarwal  
 Alan H. Cookson  
 Chester C. Taylor

Kim Breitfelder (1995-present), *IEEE Std 100 Editor*  
 Stephen Huffman (1993-1995), *IEEE Std 100 Editor*

Assistance was provided by the IEEE Standards editorial staff.

## How to use this dictionary

The terms defined in this dictionary are listed in *letter-by-letter* alphabetical order. Spaces are ignored in this style of alphabetization, so *cable value* will come before *cab signal*. Descriptive categories associated with the term in earlier editions of IEEE Std 100 will follow the term in parentheses. New categories appear after the definitions (see Categories, below), followed by the designation of the standard or standards that include the definition. If a standard designation is followed by the letter s, it means that edition of the standard was superseded by a newer revision and the term was not included in the revision. If a designation is followed by the letter w, it means that edition of the standard was withdrawn and not replaced by a revision. A bracketed number refers to the non-IEEE standard sources given in the back of the book.

Acronyms and abbreviations are no longer listed in a separate section in the dictionary; rather, they are incorporated alphabetically with other terms. Each acronym or abbreviation refers to its expanded term, where it is defined. Acronyms and abbreviations for which no definition was included in past editions have been deleted from this edition of IEEE Std 100.

Abstracts of the current set of approved IEEE standards are provided in the back of the book. It should be noted that updated information about IEEE standards can be obtained at any time from the IEEE Standards World Wide Web site at <http://standards.ieee.org/>.

## Categories

The category abbreviations that are used in this edition of IEEE Std 100 are defined below. This information is provided to help elucidate the context of the definition. Older terms for which no category could be found have had the category "Std100" assigned to them. Note that terms from sources other than IEEE standards, such as the National Electrical Code® (NEC®) or the National Fire Protection Association, may not be from the most recent editions; the reader is cautioned to check the latest editions of all sources for the most up-to-date terminology.

in such sound. *Note:* In practice, crosstalk may be measured either by the volume of the overheard sounds or by the magnitude of the coupling between the disturbed and the disturbing channels. In the latter case, to specify the volume of the overheard sounds, the volume in the disturbing channel must also be given. (PE) 599-1985w

(3) Unwanted electric signals injected into a circuit by stray coupling. (ELM) C12.1-1981

(4) (instrumentation and control equipment grounding in generating stations) The noise or extraneous signal caused by ac or dc pulse-type signals in adjacent circuits. (PE) 1050-1996

(5) (telecommunications) Undesired energy appearing in one path as a result of coupling from another path. Crosstalk is further classified as near-end, far-end, intelligible, or unintelligible. (COM) 1007-1991

(6) (telecommunications) Unwanted coupling between any two paths through the switching system. There are two situations of interest. In the first the crosstalk coupling loss of the system is linear; that is it is independent of applied disturbing signal level and is unaffected by other things, such as circuit noise. The other situation of interest arises with a digital switching system where the crosstalk coupling loss is nonlinear and is a function of applied disturbing signal level circuit noise, and encoder bias. Such systems will, under some conditions of circuit noise and coder bias, display crosstalk coupling loss that tends to decrease with decreasing disturbing signal level. *See also:* crosstalk coupling loss; equal-level crosstalk coupling loss. (COM) 973-1990w

(7) (multichannel) The ratio of the signal induced in one channel to a common signal applied to all other channels. (IM) 1057-1994

(8) The noise or extraneous signal caused by ac or pulse-type signals in adjacent circuits (measurement of power frequency magnetic fields). (PE/T&D) 1308-1994, 644-1994

(9) An electromagnetic field in the space surrounding a cable circuit created by an electrical signal. This field induces currents and electromotive forces in other circuits located close enough to the disturbing cable circuit to be affected. (PE) 1143-1994

(10) (A) A type of noise characterized by unwanted coupling of a signal or the interaction of signals on two adjacent channels. *See also:* near end crosstalk. (B) Undesired energy appearing in one signal path as a result of coupling from other signal paths. (C) 610.7-1995

(11) Crosstalk is undesired energy appearing in one signal path as a result of coupling from other signal paths. (C/LM) 8802-5-1995

**crosstalk coupling (crosstalk loss)** Cross coupling between speech communication channels or their component parts. *Note:* Crosstalk coupling is measured between specified points of the disturbing and disturbed circuits and is preferably expressed in decibels. *See also:* coupling. (EEC/PE) [119]

**crosstalk coupling loss (telecommunications)** The loss of the crosstalk path. *See also:* crosstalk. (COM) 973-1990w

**crosstalk, electron beam (charge-storage tubes)** Any spurious output signal that arises from scanning or from the input of information. *See also:* charge-storage tube. (ED) 158-1962w

**crosstalk loss** *See:* crosstalk coupling.

**crosstalk unit** Crosstalk coupling is sometimes expressed in crosstalk units through the relation

$$\text{Crosstalk units} = 10^{[6 - (L/20)]}$$

where  $L$  = crosstalk coupling in decibels. *Note:* For two circuits of equal impedance, the number of crosstalk units expresses the current in the disturbed circuit as millionths of the current in the disturbing circuit. *See also:* coupling. (EEC/PE) [119]

**crowbar** A protective circuit in a power distribution circuit that rapidly shorts the output voltage to ground when an over-voltage or other error condition occurs. (C) 610.10-1994

**CR-RC shaping (1) (charged-particle detectors) (germanium gamma-ray detectors) (x-ray energy spectrometers)** The pulse shaping present in an amplifier that has a simple high-pass filter consisting of a capacitor and a resistor together with a simple low-pass filter, separated by impedance isolation. (Pulse shaping in such an amplifier cuts off at 6 dB (decibels) per octave at both ends of the band.) (NPS) 301-1976s, 325-1986s, 759-1984r

(2) (semiconductor charged-particle detectors) In an amplifier, the pulse shaping produced by a single-section high-pass network followed by a single-section low-pass network, with the two networks separated by an isolating stage and with both networks having the same time constant. (NPS) 300-1988r

**CRS** *See:* configuration report server.

**CRT** *See:* cathode-ray tube.

**CRT display device** A display device that displays data onto a phosphor coated display screen using controlled electron beams within a CRT. *Note:* Raster display devices and random-scan display devices are two major categories of CRT display devices. *See also:* dark trace tube display device; penetration CRT display device; raster display device; storage tube display device. (C) 610.10-1994

**crude metal** Metal that contains impurities in sufficient quantities to make it unsuitable for specified purposes or that contains more valuable metals in sufficient quantities to justify their recovery. *See also:* electrorefining. (EEC/PE) [119]

**crust** A layer of solidified electrolyte. *See also:* fused electrolyte. (EEC/PE) [119]

**CRV** *See:* Code Rule Violation.

**cryogenics (1) (general)** The study and use of devices utilizing properties of materials near absolute-zero temperature. (C) [20], [85]

(2) (laser maser) The branch of physics dealing with very low temperatures. (LEO) 586-1980w

(3) A branch of technology concerned with devices that make use of the properties assumed by materials at temperatures near absolute zero. (C) 610.2-1987

**cryogenic storage** A type of storage that uses the superconductive and magnetic properties of certain materials at temperatures near absolute zero. (C) 610.10-1994

**cryotron (1)** A superconductive device in which current in one or more input circuits magnetically controls the superconducting-to-normal transition in one or more output circuits, provided the current in each output circuit is less than its critical value. *See also:* superconductivity. (ED) [46]

(2) A device that makes uses of the effects of extremely low temperatures on conductive materials such that small magnetic field changes can control large current changes. (C) 610.10-1994

**cryptographic checkvalue** Information that is derived by performing a cryptographic transformation on the data unit. *See also:* cryptography. (C/LM) 802.10-1992

**cryptography** The discipline embodying principles, means, and methods for the transformation of data in order to hide its information content, prevent its undetected modification, and/or prevent its unauthorized use. (C/LM) 802.10-1992

**crystal (A) (communication practice)** A piezoelectric crystal. (B) (communication practice) A piezoelectric crystal plate. (C) (communication practice) A crystal rectifier. (EEC/PE) [119]

**crystal-controlled oscillator** *See:* crystal oscillator.

**crystal diode** A rectifying element comprising a semiconducting crystal having two terminals designed for use in circuits in a manner analogous to that of electron-tube diodes. *See also:* rectifier. (EEC/PE) [119]

**crystal loudspeaker (piezoelectric loudspeaker)** A loudspeaker in which the mechanical displacements are produced by piezoelectric action. (EEC/PE) [119]

**crystal microphone (piezoelectric microphone)** A microphone that depends for its operation of the generation of an

**crowbar** A protective circuit in a power distribution circuit that rapidly shorts the output voltage to ground when an over-voltage or other error condition occurs. (C) 610.10-1994

**crystal microphone (piezoelectric microphone)** A microphone that depends for its operation of the generation of an



inside the building below such area, that has no landing opening into the building at its upper limit of travel, and that is not used to carry automobiles. *See also*: elevator.

(EEC/PE) [119]

**side-wall pressure** The crushing force exerted on a cable during installation. (NESC) C2-1997

**sideways sum (mathematics of computing)** A sum obtained by adding the digits of a numeral without regard to position or significance. (C) 1084-1986w

**siemens (metric practice)** The electric conductance of a conductor in which a current of one ampere is produced by an electric potential difference of one volt.

(QUL) 268-1982s

**sievert (metric practice)** The dose equivalent when the absorbed dose of ionizing radiation multiplied by the dimensionless factors  $Q$  (quality factor) and  $N$  (product of any other multiplying factors) stipulated by the International Commission on Radiological Protection is one joule per kilogram.

(QUL) 268-1982s

**sifting sort** *See*: bubble sort.

**sigma** The term sigma designates a group of telephone wires, usually the majority or all wires of a line, that is treated as a unit in the computation of noise or in arranging connections to ground for the measurement of noise or current balance ratio. (EEC/PE) [119]

**sign (1) (power or energy)** Positive, if the actual direction of energy flow agrees with the stated or implied reference direction; negative, if the actual direction is opposite to the reference direction. *See also*: network analysis.

(Std100) 270-1966w

**(2) (test, measurement, and diagnostic equipment)** The symbol that distinguishes positive from negative numbers.

(MIL) [2]

**(3) *See also*: electric sign.**

(NESC) [86]

**signal (1) (microcomputer system bus) (signals and paths)** The physical representation of data. (C/MM) 796-1983r

**(2) (signals and paths, 696 interface devices)** The physical representation which conveys data from one point to another. For the purpose of IEEE Std 696-1983, this applies to digital electrical signals only. (C/MM) 696-1983w

**(3) (A) (data transmission)** A visual, audible or other indication used to convey information. **(B) (data transmission)** The intelligence, message or effect to be conveyed over a communication system. **(C) (data transmission)** A signal wave; the physical embodiment of a message.

(PE) 599-1985w

**(4) (overhead-power-line corona and radio noise)** The intelligence, message, or effect conveyed over a communication system. (PE/T&D) 539-1990

**(5) (programmable instrumentation)** The physical representation of information. *Note*: For the purposes of IEEE Std 488.1-1987, this term refers to digital electrical signals only.

(IM) 488.1-1987r

**(6) (computers)** The event or phenomenon that conveys data from one point to another. (C) [20], [85]

**(7)** Information about a variable that can be transmitted in a system. (IA) [60]

**(8) (telephone switching systems)** An audible, visual or other indication of information. (C) [85]

**(9)** A phenomenon (visual, audible, or otherwise) used to convey information. The signal is often coded, such as a modulated waveform, so that it requires decoding to be intelligible. (CAS) [13]

**(10) (SBX bus)** The physical representation of a logical value. (C/MM) 959-1988r

**(11) (STEBus)** The physical representation of data. (C/MM) 1000-1987r

**(12)** Any communication between message-based devices consisting of a write to a signal register. (C/MM) 1155-1992

**(13)** A measurable quantity (e.g., a voltage) which varies in time in order to transmit information. A signal propagates

along a wire or an optic fiber. It is interpreted as a sequence of bits, which is grouped into a sequence of characters by the character layer of the protocol stack. Signals are generated by a link output and are absorbed by a link input.

(BA/C) 1355-1995

**(14)** In networking, an electrical pulse that conveys information through a transmission medium. *See also*: analog signal; baseband signaling; broadband signaling; digital signal; out-of-band signaling. (C) 610.7-1995

**(15) (A)** A variation of a physical quantity, used to convey data. **(B)** A time-dependent value attached to a physical phenomenon and conveying data. (C) 610.10-1994

**(16)** A mechanism by which a process may be notified of, or affected by, an event occurring in the system. Examples of such events include hardware exceptions and specific actions by processes or threads. The term *signal* is also used to refer to the event itself.

(C/PA) 1003.5-1992, 1003.5b-1995, 9945-1-1996, 9945-2-1993

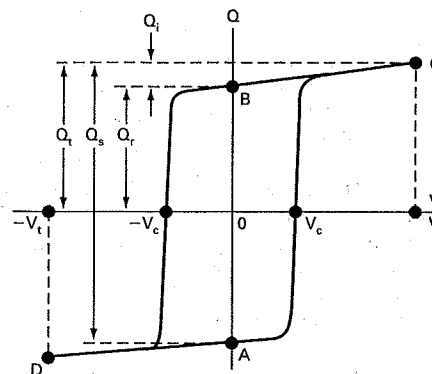
**signal, actuating** *See*: actuating signal.

**signal aspect** The appearance of a fixed signal conveying an indication as viewed from the direction of an approaching train: the appearance of a cab signal conveying an indication as viewed by an observer in the cab. (EEC/PE) [119]

**signal assertion** **A)** The act of driving a signal to the true state. **B)** The act of driving a bus of signals to the correct pattern of ones and zeros. (BA/C) 1496-1993

**signal back light** A light showing through a small opening in the back of an electrically lighted signal, used for checking the operation of the signal lamp. (EEC/PE) [119]

**signal charge** The charge that flows when the condition of the device is changed from that of zero applied voltage (after having previously been saturated with either a positive or negative voltage) to at least that voltage necessary to saturate in the reverse sense. *Note*: The signal charge  $Q_s$  equals the sum of  $Q_r$  and  $Q_t$ , as illustrated in the corresponding figure. It is dependent on the magnitude of the applied voltage, which should be specified in describing this characteristic of ferroelectric devices. *See also*: ferroelectric domain.



Hysteresis loop for a ferroelectric device.

**signal charge**

(UFFC) [180]w

**signal circuit (1)** Any electric circuit that supplies energy to an appliance that gives a recognizable signal. Such circuits include circuits for door bells, buzzers, code-calling systems, signal lights, and the like. *See also*: appliance. (NESC) [86]

**(2) (protective relay system)** Any circuit other than input voltage circuits, input current circuits, power supply circuits, or those circuits that directly or indirectly control power circuit breaker operation. (PE/SWG) C37.100-1992, C37.90-1978s

**(3) (protective relay system)** Any circuit other than an input voltage circuit, input current circuit, power supply circuit, or an output circuit. (PE) C37.90.1-1989r

signal c

signal c

tweet

signa

signal c

ment

to an

signal c

curre

tons,

incid

signal c

dress

of the

off c

signal c

signal c

work

purp

signal,

signal c

in w

same

(2) c

tance

signal

of si

signal

sign

pres

signal

sign

signal

of a

code

sign

(2)

half

Log

signal,

mag

the

(2) c

age

two

The

corr

(3)

sult

corr

*See*

signal

trol

poi

(2)

fro

ure

# **FIPS PUB 186-4**

---

## **FEDERAL INFORMATION PROCESSING STANDARDS PUBLICATION**

### **Digital Signature Standard (DSS)**

**CATEGORY: COMPUTER SECURITY**

**SUBCATEGORY: CRYPTOGRAPHY**

---

Information Technology Laboratory  
National Institute of Standards and Technology  
Gaithersburg, MD 20899-8900

Issued July 2013



**U.S. Department of Commerce**

*Cameron F. Kerry, Acting Secretary*

**National Institute of Standards and Technology**

*Patrick D. Gallagher, Under Secretary of Commerce for Standards and Technology and Director*

## **FOREWORD**

The Federal Information Processing Standards Publication Series of the National Institute of Standards and Technology (NIST) is the official series of publications relating to standards and guidelines adopted and promulgated under the provisions of the Federal Information Security Management Act (FISMA) of 2002.

Comments concerning FIPS publications are welcomed and should be addressed to the Director, Information Technology Laboratory, National Institute of Standards and Technology, 100 Bureau Drive, Stop 8900, Gaithersburg, MD 20899-8900.

Charles Romine, Director  
Information Technology Laboratory

## **Abstract**

This Standard specifies a suite of algorithms that can be used to generate a digital signature. Digital signatures are used to detect unauthorized modifications to data and to authenticate the identity of the signatory. In addition, the recipient of signed data can use a digital signature as evidence in demonstrating to a third party that the signature was, in fact, generated by the claimed signatory. This is known as non-repudiation, since the signatory cannot easily repudiate the signature at a later time.

*Key words:* computer security, cryptography, digital signatures, Federal Information Processing Standards, public key cryptography.

## Federal Information Processing Standards Publication 186-4

July 2013

### Announcing the DIGITAL SIGNATURE STANDARD (DSS)

Federal Information Processing Standards Publications (FIPS PUBS) are issued by the National Institute of Standards and Technology (NIST) after approval by the Secretary of Commerce pursuant to Section 5131 of the Information Technology Management Reform Act of 1996 (Public Law 104-106), and the Computer Security Act of 1987 (Public Law 100-235).

**1. Name of Standard:** Digital Signature Standard (DSS) (FIPS 186-4).

**2. Category of Standard:** Computer Security. **Subcategory.** Cryptography.

**3. Explanation:** This Standard specifies algorithms for applications requiring a digital signature, rather than a written signature. A digital signature is represented in a computer as a string of bits. A digital signature is computed using a set of rules and a set of parameters that allow the identity of the signatory and the integrity of the data to be verified. Digital signatures may be generated on both stored and transmitted data.

Signature generation uses a private key to generate a digital signature; signature verification uses a public key that corresponds to, but is not the same as, the private key. Each signatory possesses a private and public key pair. Public keys may be known by the public; private keys are kept secret. Anyone can verify the signature by employing the signatory's public key. Only the user that possesses the private key can perform signature generation.

A hash function is used in the signature generation process to obtain a condensed version of the data to be signed; the condensed version of the data is often called a message digest. The message digest is input to the digital signature algorithm to generate the digital signature. The hash functions to be used are specified in the Secure Hash Standard (SHS), FIPS 180. FIPS **approved** digital signature algorithms **shall** be used with an appropriate hash function that is specified in the SHS.

The digital signature is provided to the intended verifier along with the signed data. The verifying entity verifies the signature by using the claimed signatory's public key and the same hash function that was used to generate the signature. Similar procedures may be used to generate and verify signatures for both stored and transmitted data.

**4. Approving Authority:** Secretary of Commerce.

**5. Maintenance Agency:** Department of Commerce, National Institute of Standards and Technology, Information Technology Laboratory, Computer Security Division.

**6. Applicability:** This Standard is applicable to all Federal departments and agencies for the protection of sensitive unclassified information that is not subject to section 2315 of Title 10, United States Code, or section 3502 (2) of Title 44, United States Code. This Standard **shall** be used in designing and implementing public key-based signature systems that Federal departments and agencies operate or that are operated for them under contract. The adoption and use of this Standard is available to private and commercial organizations.

**7. Applications:** A digital signature algorithm allows an entity to authenticate the integrity of signed data and the identity of the signatory. The recipient of a signed message can use a digital signature as evidence in demonstrating to a third party that the signature was, in fact, generated by the claimed signatory. This is known as non-repudiation, since the signatory cannot easily repudiate the signature at a later time. A digital signature algorithm is intended for use in electronic mail, electronic funds transfer, electronic data interchange, software distribution, data storage, and other applications that require data integrity assurance and data origin authentication.

**8. Implementations:** A digital signature algorithm may be implemented in software, firmware, hardware or any combination thereof. NIST has developed a validation program to test implementations for conformance to the algorithms in this Standard. Information about the validation program is available at <http://csrc.nist.gov/cryptval>. Examples for each digital signature algorithm are available at <http://csrc.nist.gov/groups/ST/toolkit/examples.html>.

Agencies are advised that digital signature key pairs **shall not** be used for other purposes.

**9. Other Approved Security Functions:** Digital signature implementations that comply with this Standard **shall** employ cryptographic algorithms, cryptographic key generation algorithms, and key establishment techniques that have been approved for protecting Federal government sensitive information. Approved cryptographic algorithms and techniques include those that are either:

- a. specified in a Federal Information Processing Standard (FIPS),
- b. adopted in a FIPS or a NIST Recommendation, or
- c. specified in the list of approved security functions for FIPS 140.

**10. Export Control:** Certain cryptographic devices and technical data regarding them are subject to Federal export controls. Exports of cryptographic modules implementing this Standard and technical data regarding them must comply with these Federal regulations and be licensed by the Bureau of Industry and Security of the U.S. Department of Commerce. Information about export regulations is available at: <http://www.bis.doc.gov>.

**11. Patents:** The algorithms in this Standard may be covered by U.S. or foreign patents.

**12. Implementation Schedule:** This Standard becomes effective immediately upon approval by the Secretary of Commerce. A transition strategy for validating algorithms and cryptographic modules will be posted on NIST's Web page at <http://csrc.nist.gov/groups/STM/cmvp/index.html> under Notices. The transition plan addresses the transition by Federal agencies from modules tested and validated for compliance to previous versions of this Standard to modules tested and validated for compliance to FIPS 186-4 under the Cryptographic Module Validation Program. The transition plan allows Federal agencies and vendors to make a smooth transition to FIPS 186-4.

**13. Specifications:** Federal Information Processing Standard (FIPS) 186-4 Digital Signature Standard (affixed).

**14. Cross Index:** The following documents are referenced in this Standard. Unless a specific version or date is indicated with the document number, the latest version of the given document is intended as the reference.

- a. FIPS PUB 140, Security Requirements for Cryptographic Modules.
- b. FIPS PUB 180 Secure Hash Standard.
- c. ANS X9.31-1998, Digital Signatures Using Reversible Public Key Cryptography for the Financial Services Industry (rDSA).
- d. ANS X9.62-2005, Public Key Cryptography for the Financial Services Industry: The Elliptic Curve Digital Signature Algorithm (ECDSA).
- e. ANS X9.80, Prime Number Generation, Primality Testing and Primality Certificates.
- f. Public Key Cryptography Standard (PKCS) #1, RSA Encryption Standard.
- g. Special Publication (SP) 800-57, Recommendation for Key Management.
- h. Special Publication (SP) 800-89, Recommendation for Obtaining Assurances for Digital Signature Applications.
- i. Special Publication (SP) 800-90A, Recommendation for Random Number Generation Using Deterministic Random Bit Generators.
- j. Special Publication (SP) 800-102, Recommendation for Digital Signature Timeliness.
- k. Special Publication (SP) 800-131A, Transitions: Recommendation for Transitioning the Use of Cryptographic Algorithms and Key Lengths.
- l. IEEE Std. 1363-2000, Standard Specifications for Public Key Cryptography.

**15. Qualifications:** The security of a digital signature system is dependent on maintaining the secrecy of the signatory's private keys. Signatories **shall**, therefore, guard against the disclosure of their private keys. While it is the intent of this Standard to specify general security requirements for generating digital signatures, conformance to this Standard does not assure that

a particular implementation is secure. It is the responsibility of an implementer to ensure that any module that implements a digital signature capability is designed and built in a secure manner.

Similarly, the use of a product containing an implementation that conforms to this Standard does not guarantee the security of the overall system in which the product is used. The responsible authority in each agency or department **shall** assure that an overall implementation provides an acceptable level of security.

Since a standard of this nature must be flexible enough to adapt to advancements and innovations in science and technology, this Standard will be reviewed every five years in order to assess its adequacy.

**16. Waiver Procedure:** The Federal Information Security Management Act (FISMA) does not allow for waivers to Federal Information Processing Standards (FIPS) that are made mandatory by the Secretary of Commerce.

**17. Where to Obtain Copies of the Standard:** This publication is available by accessing <http://csrc.nist.gov/publications/>. Other computer security publications are available at the same web site.



## Table of Contents

<b>1. INTRODUCTION .....</b>	<b>1</b>
<b>2. GLOSSARY OF TERMS, ACRONYMS AND MATHEMATICAL SYMBOLS .....</b>	<b>2</b>
2.1 TERMS AND DEFINITIONS .....	2
2.2 ACRONYMS .....	5
2.3 MATHEMATICAL SYMBOLS .....	6
<b>3. GENERAL DISCUSSION .....</b>	<b>9</b>
3.1 INITIAL SETUP .....	11
3.2 DIGITAL SIGNATURE GENERATION .....	12
3.3 DIGITAL SIGNATURE VERIFICATION AND VALIDATION .....	13
<b>4 THE DIGITAL SIGNATURE ALGORITHM (DSA) .....</b>	<b>15</b>
4.1 DSA PARAMETERS .....	15
4.2 SELECTION OF PARAMETER SIZES AND HASH FUNCTIONS FOR DSA .....	15
4.3 DSA DOMAIN PARAMETERS .....	16
4.3.1 Domain Parameter Generation .....	17
4.3.2 Domain Parameter Management .....	17
4.4 KEY PAIRS .....	17
4.4.1 DSA Key Pair Generation .....	17
4.4.2 Key Pair Management .....	18
4.5 DSA PER-MESSAGE SECRET NUMBER .....	18
4.6 DSA SIGNATURE GENERATION .....	19
4.7 DSA SIGNATURE VERIFICATION AND VALIDATION .....	19
<b>5. THE RSA DIGITAL SIGNATURE ALGORITHM .....</b>	<b>22</b>
5.1 RSA KEY PAIR GENERATION .....	22
5.2 KEY PAIR MANAGEMENT .....	23
5.3 ASSURANCES .....	23
5.4 ANS X9.31 .....	24
5.5 PKCS #1 .....	24
<b>6. THE ELLIPTIC CURVE DIGITAL SIGNATURE ALGORITHM (ECDSA) .....</b>	<b>26</b>
6.1 ECDSA DOMAIN PARAMETERS .....	26
6.1.1 Domain Parameter Generation .....	26
6.1.2 Domain Parameter Management .....	28
6.2 PRIVATE/PUBLIC KEYS .....	28
6.2.1 Key Pair Generation .....	29
6.2.2 Key Pair Management .....	29
6.3 SECRET NUMBER GENERATION .....	29
6.4 ECDSA DIGITAL SIGNATURE GENERATION AND VERIFICATION .....	29
6.5 ASSURANCES .....	30
<b>APPENDIX A: GENERATION AND VALIDATION OF FFC DOMAIN PARAMETERS .....</b>	<b>31</b>



A.1	GENERATION OF THE FFC PRIMES $P$ AND $Q$ .....	31
A.1.1	Generation and Validation of Probable Primes .....	31
A.1.2	Construction and Validation of the Provable Primes $p$ and $q$ .....	36
A.2	GENERATION OF THE GENERATOR $G$ .....	41
A.2.1	Unverifiable Generation of the Generator $g$ .....	41
A.2.2	Assurance of the Validity of the Generator $g$ .....	42
A.2.3	Verifiable Canonical Generation of the Generator $g$ .....	42
A.2.4	Validation Routine when the Canonical Generation of the Generator $g$ Routine Was Used.....	44
<b>APPENDIX B: KEY PAIR GENERATION .....</b>		<b>46</b>
B.1	FFC KEY PAIR GENERATION.....	46
B.1.1	Key Pair Generation Using Extra Random Bits .....	46
B.1.2	Key Pair Generation by Testing Candidates.....	47
B.2	FFC PER-MESSAGE SECRET NUMBER GENERATION .....	48
B.2.1	Per-Message Secret Number Generation Using Extra Random Bits .....	49
B.2.2	Per-Message Secret Number Generation by Testing Candidates.....	49
B.3	IFC KEY PAIR GENERATION.....	50
B.3.1	Criteria for IFC Key Pairs .....	50
B.3.2	Generation of Random Primes that are Provably Prime.....	53
B.3.3	Generation of Random Primes that are Probably Prime.....	55
B.3.4	Generation of Provable Primes with Conditions Based on Auxiliary Provable Primes ..	56
B.3.5	Generation of Probable Primes with Conditions Based on Auxiliary Provable Primes ..	58
B.3.6	Generation of Probable Primes with Conditions Based on Auxiliary Probable Primes ..	60
B.4	ECC KEY PAIR GENERATION .....	61
B.4.1	Key Pair Generation Using Extra Random Bits .....	62
B.4.2	Key Pair Generation by Testing Candidates.....	63
B.5	ECC PER-MESSAGE SECRET NUMBER GENERATION.....	64
B.5.1	Per-Message Secret Number Generation Using Extra Random Bits .....	64
B.5.2	Per-Message Secret Number Generation by Testing Candidates.....	65
<b>APPENDIX C: GENERATION OF OTHER QUANTITIES.....</b>		<b>67</b>
C.1	COMPUTATION OF THE INVERSE VALUE .....	67
C.2	CONVERSION BETWEEN BIT STRINGS AND INTEGERS.....	68
C.2.1	Conversion of a Bit String to an Integer .....	68
C.2.2	Conversion of an Integer to a Bit String .....	68
C.3	PROBABILISTIC PRIMALITY TESTS.....	69
C.3.1	Miller-Rabin Probabilistic Primality Test.....	71
C.3.2	Enhanced Miller-Rabin Probabilistic Primality Test .....	72

C.3.3 (GENERAL) LUCAS PROBABILISTIC PRIMALITY TEST .....	74
C.4 CHECKING FOR A PERFECT SQUARE .....	75
C.5 JACOBI SYMBOL ALGORITHM.....	76
C.6 SHAWE-TAYLOR RANDOM_PRIME ROUTINE.....	77
C.7 TRIAL DIVISION .....	80
C.8 SIEVE PROCEDURE .....	80
C.9 COMPUTE A PROBABLE PRIME FACTOR BASED ON AUXILIARY PRIMES .....	81
C.10 CONSTRUCT A PROVABLE PRIME (POSSIBLY WITH CONDITIONS), BASED ON CONTEMPORANEOUSLY CONSTRUCTED AUXILIARY PROVABLE PRIMES.....	83
<b>APPENDIX D: RECOMMENDED ELLIPTIC CURVES FOR FEDERAL GOVERNMENT USE .....</b>	<b>87</b>
D.1 NIST RECOMMENDED ELLIPTIC CURVES .....	87
D.1.1 Choices .....	87
D.1.2 Curves over Prime Fields.....	89
D.1.3 Curves over Binary Fields.....	92
D.2 IMPLEMENTATION OF MODULAR ARITHMETIC.....	101
D.2.1 Curve P-192.....	101
D.2.2 Curve P-224.....	102
D.2.3 Curve P-256.....	102
D.2.4 Curve P-384.....	103
D.2.5 Curve P-521 .....	104
D.3 NORMAL BASES .....	104
D.4 SCALAR MULTIPLICATION ON KOBLITZ CURVES.....	106
D.5 GENERATION OF PSEUDO-RANDOM CURVES (PRIME CASE) .....	109
D.6 VERIFICATION OF CURVE PSEUDO-RANDOMNESS (PRIME CASE) .....	110
D.7 GENERATION OF PSEUDO-RANDOM CURVES (BINARY CASE).....	111
D.8 VERIFICATION OF CURVE PSEUDO-RANDOMNESS (BINARY CASE).....	111
D.9 POLYNOMIAL BASIS TO NORMAL BASIS CONVERSION .....	112
D.10 NORMAL BASIS TO POLYNOMIAL BASIS CONVERSION .....	113
<b>APPENDIX E: A PROOF THAT <math>V = R</math> IN THE DSA.....</b>	<b>115</b>
<b>APPENDIX F: CALCULATING THE REQUIRED NUMBER OF ROUNDS OF TESTING USING THE MILLER-RABIN PROBABILISTIC PRIMALITY TEST .....</b>	<b>117</b>
F.1 THE REQUIRED NUMBER OF ROUNDS OF THE MILLER-RABIN PRIMALITY TESTS .....	117
F.2 GENERATING DSA PRIMES.....	118
F.3 GENERATING PRIMES FOR RSA SIGNATURES .....	119
<b>APPENDIX G: REFERENCES .....</b>	<b>121</b>

## **Federal Information Processing Standards Publication 186-4**

**July 2013**

### **Specifications for the DIGITAL SIGNATURE STANDARD (DSS)**

#### **1. Introduction**

This Standard defines methods for digital signature generation that can be used for the protection of binary data (commonly called a message), and for the verification and validation of those digital signatures. Three techniques are approved.

- (1) The Digital Signature Algorithm (DSA) is specified in this Standard. The specification includes criteria for the generation of domain parameters, for the generation of public and private key pairs, and for the generation and verification of digital signatures.
- (2) The RSA digital signature algorithm is specified in American National Standard (ANS) X9.31 and Public Key Cryptography Standard (PKCS) #1. FIPS 186-4 approves the use of implementations of either or both of these standards and specifies additional requirements.
- (3) The Elliptic Curve Digital Signature Algorithm (ECDSA) is specified in ANS X9.62. FIPS 186-4 approves the use of ECDSA and specifies additional requirements. Recommended elliptic curves for Federal Government use are provided herein.

This Standard includes requirements for obtaining the assurances necessary for valid digital signatures. Methods for obtaining these assurances are provided in NIST Special Publication (SP) 800-89, *Recommendation for Obtaining Assurances for Digital Signature Applications*.

## 2. Glossary of Terms, Acronyms and Mathematical Symbols

### 2.1 Terms and Definitions

Approved	FIPS-approved and/or NIST-recommended. An algorithm or technique that is either 1) specified in a FIPS or NIST Recommendation, or 2) adopted in a FIPS or NIST Recommendation or 3) specified in a list of NIST approved security functions.
Assurance of domain parameter validity	Confidence that the domain parameters are arithmetically correct.
Assurance of possession	Confidence that an entity possesses a private key and any associated keying material.
Assurance of public key validity	Confidence that the public key is arithmetically correct.
Bit string	An ordered sequence of 0's and 1's. The leftmost bit is the most significant bit of the string. The rightmost bit is the least significant bit of the string.
Certificate	A set of data that uniquely identifies a key pair and an owner that is authorized to use the key pair. The certificate contains the owner's public key and possibly other information, and is digitally signed by a Certification Authority (i.e., a trusted party), thereby binding the public key to the owner.
Certification Authority (CA)	The entity in a Public Key Infrastructure (PKI) that is responsible for issuing certificates and exacting compliance with a PKI policy.
Claimed signatory	From the verifier's perspective, the claimed signatory is the entity that purportedly generated a digital signature.
Digital signature	The result of a cryptographic transformation of data that, when properly implemented, provides a mechanism for verifying origin authentication, data integrity and signatory non-repudiation.
Domain parameter seed	A string of bits that is used as input for a domain parameter generation or validation process.
Domain parameters	Parameters used with cryptographic algorithms that are usually common to a domain of users. A DSA or ECDSA cryptographic key pair is associated with a specific set of domain parameters.

Entity	An individual (person), organization, device or process. Used interchangeably with “party”.
Equivalent process	Two processes are equivalent if, when the same values are input to each process (either as input parameters or as values made available during the process or both), the same output is produced.
Hash function	<p>A function that maps a bit string of arbitrary length to a fixed length bit string. Approved hash functions are specified in FIPS 180 and are designed to satisfy the following properties:</p> <ol style="list-style-type: none"><li>1. (One-way) It is computationally infeasible to find any input that maps to any new pre-specified output, and</li><li>2. (Collision resistant) It is computationally infeasible to find any two distinct inputs that map to the same output.</li></ol>
Hash value	See “message digest”.
Intended signatory	An entity that intends to generate digital signatures in the future.
Key	<p>A parameter used in conjunction with a cryptographic algorithm that determines its operation. Examples applicable to this Standard include:</p> <ol style="list-style-type: none"><li>1. The computation of a digital signature from data, and</li><li>2. The verification of a digital signature.</li></ol>
Key pair	A public key and its corresponding private key.
Message	The data that is signed. Also known as “signed data” during the signature verification and validation process.
Message digest	The result of applying a hash function to a message. Also known as “hash value”.
Non-repudiation	A service that is used to provide assurance of the integrity and origin of data in such a way that the integrity and origin can be verified and validated by a third party as having originated from a specific entity in possession of the private key (i.e., the signatory).
Owner	A key pair owner is the entity that is authorized to use the private key of a key pair.
Party	An individual (person), organization, device or process. Used interchangeably with “entity”.
Per-message secret number	A secret random number that is generated prior to the generation of each digital signature.

Public Key Infrastructure (PKI)	A framework that is established to issue, maintain and revoke public key certificates.
Prime number generation seed	A string of random bits that is used to determine a prime number with the required characteristics.
Private key	A cryptographic key that is used with an asymmetric (public key) cryptographic algorithm. For digital signatures, the private key is uniquely associated with the owner and is not made public. The private key is used to compute a digital signature that may be verified using the corresponding public key.
Probable prime	An integer that is believed to be prime, based on a probabilistic primality test. There should be no more than a negligible probability that the so-called probable prime is actually composite.
Provable prime	An integer that is either constructed to be prime or is calculated to be prime using a primality-proving algorithm.
Pseudorandom	A process or data produced by a process is said to be pseudorandom when the outcome is deterministic, yet also effectively random as long as the internal action of the process is hidden from observation. For cryptographic purposes, “effectively” means “within the limits of the intended security strength.”
Public key	A cryptographic key that is used with an asymmetric (public key) cryptographic algorithm and is associated with a private key. The public key is associated with an owner and may be made public. In the case of digital signatures, the public key is used to verify a digital signature that was signed using the corresponding private key.
Security strength	A number associated with the amount of work (that is, the number of operations) that is required to break a cryptographic algorithm or system. Sometimes referred to as a security level.
<b>Shall</b>	Used to indicate a requirement of this Standard.
<b>Should</b>	Used to indicate a strong recommendation, but not a requirement of this Standard.
Signatory	The entity that generates a digital signature on data using a private key.
Signature generation	The process of using a digital signature algorithm and a private key to generate a digital signature on data.

Signature validation	The (mathematical) verification of the digital signature and obtaining the appropriate assurances (e.g., public key validity, private key possession, etc.).
Signature verification	The process of using a digital signature algorithm and a public key to verify a digital signature on data.
Signed data	The data or message upon which a digital signature has been computed. Also, see “message”.
Subscriber	An entity that has applied for and received a certificate from a Certificate Authority.
Trusted third party (TTP)	An entity other than the owner and verifier that is trusted by the owner or the verifier or both. Sometimes shortened to “trusted party”.
Verifier	The entity that verifies the authenticity of a digital signature using the public key.

## **2.2 Acronyms**

ANS	American National Standard.
CA	Certification Authority.
DSA	Digital Signature Algorithm; specified in this Standard.
ECDSA	Elliptic Curve Digital Signature Algorithm; specified in ANSI X9.62.
FIPS	Federal Information Processing Standard.
NIST	National Institute of Standards and Technology.
PKCS	Public Key Cryptography Standard.
PKI	Public Key Infrastructure.
RBG	Random Bit Generator.
RSA	Algorithm developed by Rivest, Shamir and Adleman; specified in ANSI X9.31 and PKCS #1.
SHA	Secure Hash Algorithm; specified in FIPS 180.
SP	NIST Special Publication
TTP	Trusted Third Party.

## 2.3 Mathematical Symbols

$a \bmod n$	The unique remainder $r$ , $0 \leq r \leq (n - 1)$ , when integer $a$ is divided by the positive integer $n$ . For example, $23 \bmod 7 = 2$ .
$b \equiv a \bmod n$	There exists an integer $k$ such that $b - a = kn$ ; equivalently, $a \bmod n = b \bmod n$ .
<i>counter</i>	The counter value that results from the domain parameter generation process when the domain parameter seed is used to generate DSA domain parameters.
$d$	1. For RSA, the private signature exponent of a private key. 2. For ECDSA, the private key.
<i>domain_parameter_seed</i>	A seed used for the generation of domain parameters.
$e$	The public verification exponent of an RSA public key.
$g$	One of the DSA domain parameters; $g$ is a generator of the $q$ -order cyclic group of $GF(p)^*$ ; that is, an element of order $q$ in the multiplicative group of $GF(p)$ .
<b>GCD</b> ( $a, b$ )	Greatest common divisor of the integers $a$ and $b$ .
<b>Hash</b> ( $M$ )	The result of a hash computation (message digest or hash value) on message $M$ using an approved hash function.
<i>index</i>	A value used in the generation of $g$ to indicate its intended use (e.g., for digital signatures).
$k$	For DSA and ECDSA, a per-message secret number.
$L$	For DSA, the length of the parameter $p$ in bits.
$(L, N)$	The associated pair of length parameters for a DSA key pair, where $L$ is the length of $p$ , and $N$ is the length of $q$ .
<b>LCM</b> ( $a, b$ )	The least common multiple of the integers $a$ and $b$ .
<b>len</b> ( $a$ )	The length of $a$ in bits; the integer $L$ , where $2^{L-1} \leq a < 2^L$ .
$M$	The message that is signed using the digital signature algorithm.
$m$	For ECDSA, the degree of the finite field $GF_{2^m}$ .
$N$	For DSA, the length of the parameter $q$ in bits.

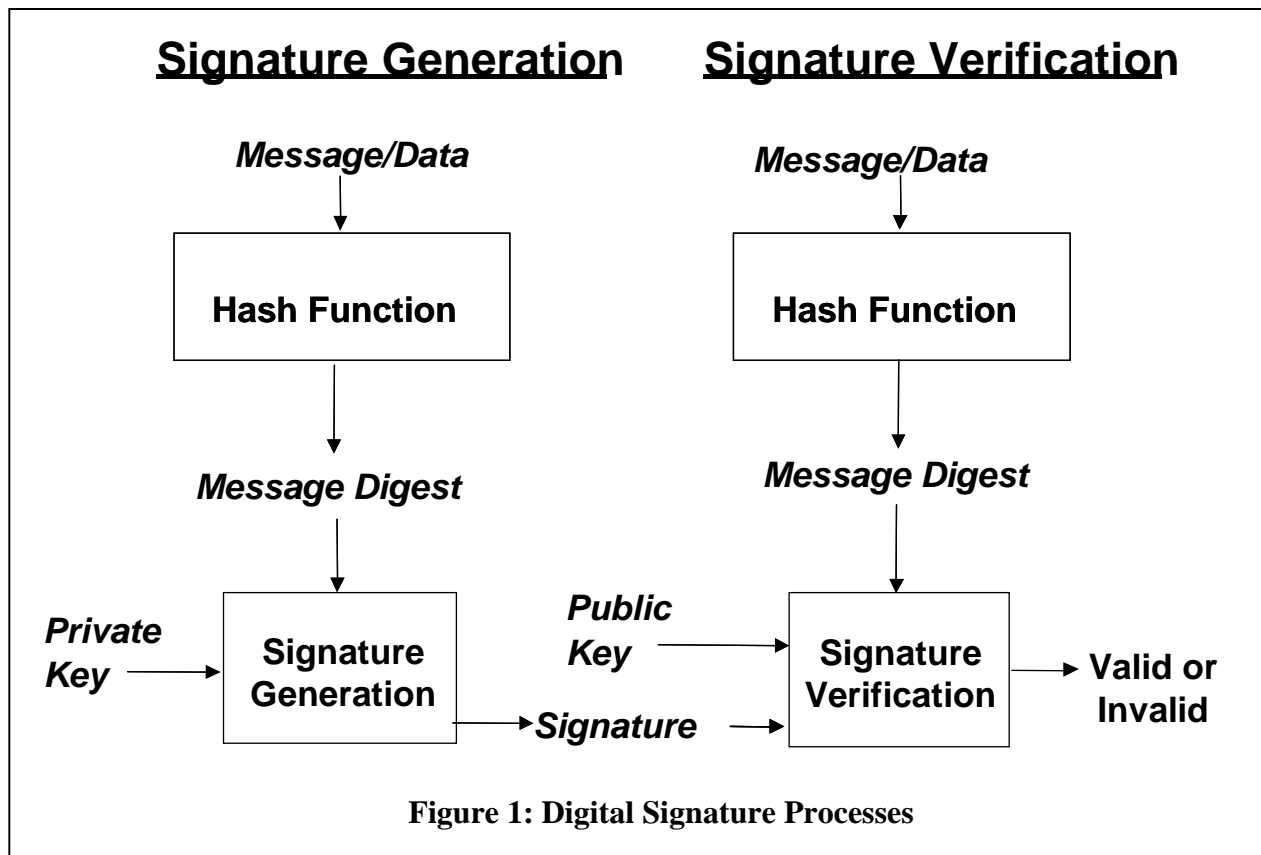


$n$	<ol style="list-style-type: none"> <li>1. For RSA, the modulus; the bit length of <math>n</math> is considered to be the key size.</li> <li>2. For ECDSA, the order of the base point of the elliptic curve; the bit length of <math>n</math> is considered to be the key size.</li> </ol>
$(n, d)$	An RSA private key, where $n$ is the modulus, and $d$ is the private signature exponent.
$(n, e)$	An RSA public key, where $n$ is the modulus, and $e$ is the public verification exponent.
$nlen$	The length of the RSA modulus $n$ in bits.
$p$	<ol style="list-style-type: none"> <li>1. For DSA, one of the DSA domain parameters; a prime number that defines the Galois Field <math>GF(p)</math> and is used as a modulus in the operations of <math>GF(p)</math>.</li> <li>2. For RSA, a prime factor of the modulus <math>n</math>.</li> </ol>
$q$	<ol style="list-style-type: none"> <li>1. For DSA, one of the DSA domain parameters; a prime factor of <math>p - 1</math>.</li> <li>2. For RSA, a prime factor of the modulus <math>n</math>.</li> </ol>
$Q$	An ECDSA public key.
$r$	One component of a DSA or ECDSA digital signature. See the definition of $(r, s)$ .
$(r, s)$	A DSA or ECDSA digital signature, where $r$ and $s$ are the digital signature components.
$s$	One component of a DSA or ECDSA digital signature. See the definition of $(r, s)$ .
$seedlen$	The length of the <i>domain_parameter_seed</i> in bits.
$SHA_x(M)$	The result when $M$ is the input to the SHA- $x$ hash function, where SHA- $x$ is specified in FIPS 180.
$x$	The DSA private key.
$y$	The DSA public key.
$\oplus$	<p>Bitwise logical “exclusive-or” on bit strings of the same length; for corresponding bits of each bit string, the result is determined as follows: <math>0 \oplus 0 = 0</math>, <math>0 \oplus 1 = 1</math>, <math>1 \oplus 0 = 1</math>, or <math>1 \oplus 1 = 0</math>.</p> <p>Example: <math>01101 \oplus 11010 = 10111</math></p>
$+$	Addition.

$*$	Multiplication.
$/$	Division.
$a \parallel b$	The concatenation of two strings $a$ and $b$ . Either $a$ and $b$ are both bit strings, or both are byte strings.
$\lceil a \rceil$	The ceiling of $a$ : the smallest integer that is greater than or equal to $a$ . For example, $\lceil 5 \rceil = 5$ , $\lceil 5.3 \rceil = 6$ , and $\lceil -2.1 \rceil = -2$ .
$\lfloor a \rfloor$	The floor of $a$ ; the largest integer that is less than or equal to $a$ . For example, $\lfloor 5 \rfloor = 5$ , $\lfloor 5.3 \rfloor = 5$ , and $\lfloor -2.1 \rfloor = -3$ .
$ a $	The absolute value of $a$ ; $ a $ is $-a$ if $a < 0$ ; otherwise, it is simply $a$ . For example, $ 2  = 2$ , and $ -2  = 2$ .
$[a, b]$	The interval of integers between and including $a$ and $b$ . For example, $[1, 4]$ consists of the integers 1, 2, 3 and 4.
$\{, a, b, \dots\}$	Used to indicate optional information.
0x	The prefix to a bit string that is represented in hexadecimal characters.

### 3. General Discussion

A digital signature is an electronic analogue of a written signature; the digital signature can be used to provide assurance that the claimed signatory signed the information. In addition, a digital signature may be used to detect whether or not the information was modified after it was signed (i.e., to detect the integrity of the signed data). These assurances may be obtained whether the data was received in a transmission or retrieved from storage. A properly implemented digital signature algorithm that meets the requirements of this Standard can provide these services.



A digital signature algorithm includes a signature generation process and a signature verification process. A signatory uses the generation process to generate a digital signature on data; a verifier uses the verification process to verify the authenticity of the signature. Each signatory has a public and private key and is the owner of that key pair. As shown in Figure 1, the private key is used in the signature generation process. The key pair owner is the only entity that is authorized to use the private key to generate digital signatures. In order to prevent other entities from claiming to be the key pair owner and using the private key to generate fraudulent signatures, the

private key must remain secret. The approved digital signature algorithms are designed to prevent an adversary who does not know the signatory's private key from generating the same signature as the signatory on a different message. In other words, signatures are designed so that they cannot be forged. A number of alternative terms are used in this Standard to refer to the signatory or key pair owner. An entity that intends to generate digital signatures in the future may be referred to as the *intended signatory*. Prior to the verification of a signed message, the signatory is referred to as the *claimed signatory* until such time as adequate assurance can be obtained of the actual identity of the signatory.

The public key is used in the signature verification process (see Figure 1). The public key need not be kept secret, but its integrity must be maintained. Anyone can verify a correctly signed message using the public key.

For both the signature generation and verification processes, the message (i.e., the signed data) is converted to a fixed-length representation of the message by means of an approved hash function. Both the original message and the digital signature are made available to a verifier.

A verifier requires assurance that the public key to be used to verify a signature belongs to the entity that claims to have generated a digital signature (i.e., the claimed signatory). That is, a verifier requires assurance that the signatory is the actual owner of the public/private key pair used to generate and verify a digital signature. A binding of an owner's identity and the owner's public key **shall** be effected in order to provide this assurance.

A verifier also requires assurance that the key pair owner actually possesses the private key associated with the public key, and that the public key is a mathematically correct key.

By obtaining these assurances, the verifier has assurance that if the digital signature can be correctly verified using the public key, the digital signature is valid (i.e., the key pair owner really signed the message). Digital signature validation includes both the (mathematical) verification of the digital signature and obtaining the appropriate assurances. The following are reasons why such assurances are required.

1. If a verifier does not obtain assurance that a signatory is the actual owner of the key pair whose public component is used to verify a signature, the problem of forging a signature is reduced to the problem of falsely claiming an identity. For example, anyone in possession of a mathematically consistent key pair can sign a message and claim that the signatory was the President of the United States. If a verifier does not require assurance that the President is actually the owner of the public key that is used to mathematically verify the message's signature, then successful signature verification provides assurance that the message has not been altered since it was signed, but does not provide assurance that the message came from the President (i.e., the verifier has assurance of the data's integrity, but source authentication is lacking).
2. If the public key used to verify a signature is not mathematically valid, the arguments used to establish the cryptographic strength of the signature algorithm may not apply.

The owner may not be the only party who can generate signatures that can be verified with that public key.

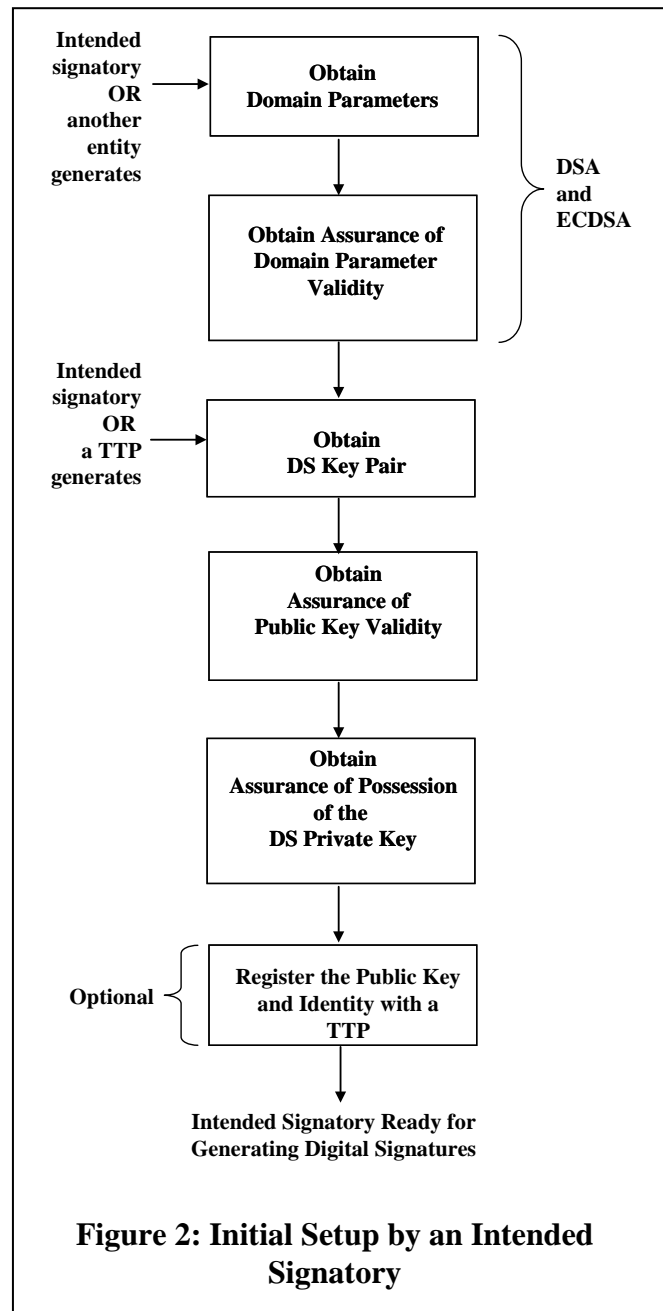
3. If a public key infrastructure cannot provide assurance to a verifier that the owner of a key pair has demonstrated knowledge of a private key that corresponds to the owner's public key, then it may be possible for an unscrupulous entity to have their identity (or an assumed identity) bound to a public key that is (or has been) used by another party. The unscrupulous entity may then claim to be the source of certain messages signed by that other party. Or, it may be possible that an unscrupulous entity has managed to obtain ownership of a public key that was chosen with the sole purpose of allowing for the verification of a signature on a specific message.

Technically, a key pair used by a digital signature algorithm could also be used for purposes other than digital signatures (e.g., for key establishment). However, a key pair used for digital signature generation and verification as specified in this Standard **shall not** be used for any other purpose. See SP 800-57 on Key Usage for further information.

A number of steps are required to enable a digital signature generation or verification capability in accordance with this Standard. All parties that generate digital signatures **shall** perform the initial setup process as discussed in Section 3.1. Digital signature generation **shall** be performed as discussed in Section 3.2. Digital signature verification and validation **shall** be performed as discussed in Section 3.3.

### 3.1 Initial Setup

Figure 2 depicts the steps that are performed prior to generating a digital



signature by an entity intending to act as a signatory.

For the DSA and ECDSA algorithms, the intended signatory **shall** first obtain appropriate domain parameters, either by generating the domain parameters itself, or by obtaining domain parameters that another entity has generated. Having obtained the set of domain parameters, the intended signatory **shall** obtain assurance of the validity of those domain parameters; approved methods for obtaining this assurance are provided in SP 800-89. Note that the RSA algorithm does not use domain parameters.

Each intended signatory **shall** obtain a digital signature key pair that is generated as specified for the appropriate digital signature algorithm, either by generating the key pair itself or by obtaining the key pair from a trusted party. The intended signatory is authorized to use the key pair and is the owner of that key pair. Note that if a trusted party generates the key pair, that party needs to be trusted not to masquerade as the owner, even though the trusted party knows the private key.

After obtaining the key pair, the intended signatory (now the key pair owner) **shall** obtain (1) assurance of the validity of the public key and (2) assurance that he/she actually possesses the associated private key. Approved methods for obtaining these assurances are provided in SP 800-89.

A digital signature verifier requires assurance of the identity of the signatory. Depending on the environment in which digital signatures are generated and verified, the key pair owner (i.e., the intended signatory) may register the public key and establish proof of identity with a mutually trusted party. For example, a certification authority (CA) could sign credentials containing an owner's public key and identity to form a certificate after being provided with proof of the owner's identity. Systems for certifying credentials and distributing certificates are beyond the scope of this Standard. Other means of establishing proof of identity (e.g., by providing identity credentials along with the public key directly to a prospective verifier) are allowed.

### 3.2 Digital Signature Generation

Figure 3 depicts the steps that are performed by an intended signatory (i.e., the entity that generates a digital signature).

Prior to the generation of a digital signature, a message digest **shall** be generated on the information to be signed using an appropriate approved hash function.

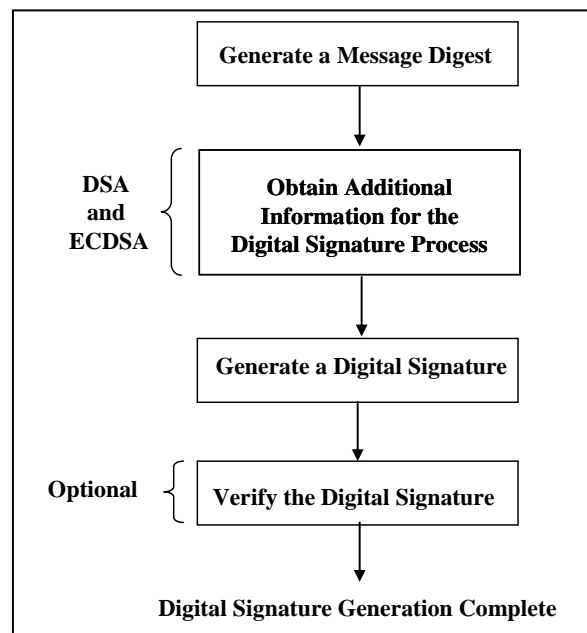


Figure 3: Digital Signature Generation

Depending on the digital signature algorithm to be used, additional information **shall** be obtained. For example, a random per-message secret number **shall** be obtained for DSA and ECDSA.

Using the selected digital signature algorithm, the signature private key, the message digest, and any other information required by the digital signature process, a digital signature **shall** be generated in accordance with this Standard.

The signatory may optionally verify the digital signature using the signature verification process and the associated public key. This optional verification serves as a final check to detect otherwise undetected signature generation computation errors; this verification may be prudent when signing a high-value message, when multiple users are expected to verify the signature, or if the verifier will be verifying the signature at a much later time.

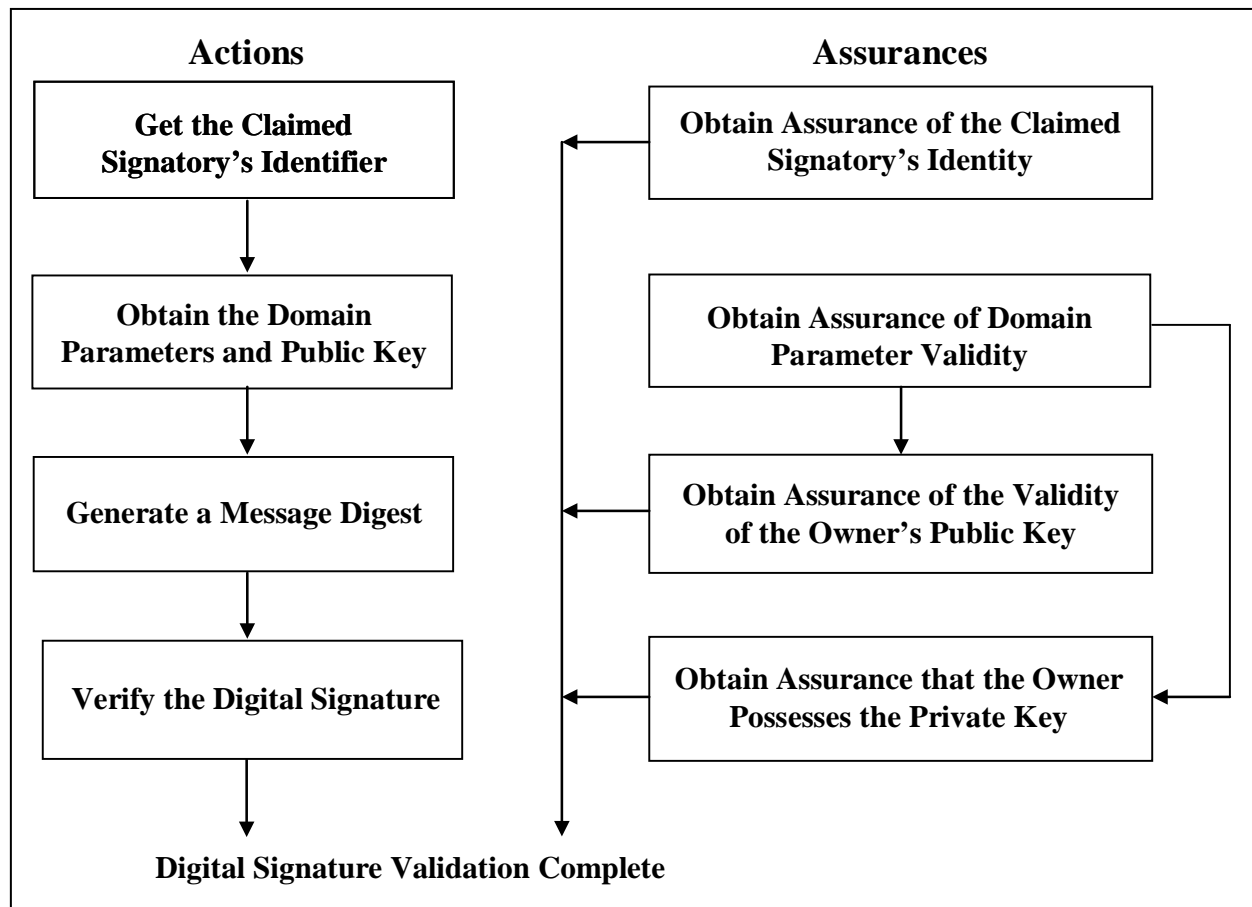
### 3.3 Digital Signature Verification and Validation

Figure 4 depicts the digital signature verification and validation process that are performed by a verifier (e.g., the intended recipient of the signed data and associated digital signature). Note that the figure depicts a successful verification and validation process (i.e., no errors are detected).

In order to verify a digital signature, the verifier **shall** obtain the public key of the claimed signatory, (usually) based on the claimed identity. If DSA or ECDSA has been used to generate the digital signature, the verifier **shall** also obtain the domain parameters. The public key and domain parameters may be obtained, for example, from a certificate created by a trusted party (e.g., a CA) or directly from the claimed signatory. A message digest **shall** be generated on the data whose signature is to be verified (i.e., not on the received digital signature) using the same hash function that was used during the digital signature generation process. Using the appropriate digital signature algorithm, the domain parameters (if appropriate), the public key and the newly computed message digest, the received digital signature is verified in accordance with this Standard. If the verification process fails, no inference can be made as to whether the data is correct, only that in using the specified public key and the specified signature format, the digital signature cannot be verified for that data.

Before accepting the verified digital signature as valid, the verifier **shall** have (1) assurance of the signatory's claimed identity, (2) assurance of the validity of the domain parameters (for DSA and ECDSA), (3) assurance of the validity of the public key, and (4) assurance that the claimed signatory actually possessed the private key that was used to generate the digital signature at the time that the signature was generated. Methods for the verifier to obtain these assurances are provided in SP 800-89. Note that assurance of domain parameter validity may have been obtained during initial setup (see Section 3.1).

If the verification and assurance processes are successful, the digital signature and signed data **shall** be considered valid. However, if a verification or assurance process fails, the digital signature **should** be considered invalid. An organization's policy **shall** govern the action to be taken for an invalid digital signature. Such policy is outside the scope of this Standard.



**Figure 4: Digital Signature Verification and Validation**



## 4 The Digital Signature Algorithm (DSA)

### 4.1 DSA Parameters

A DSA digital signature is computed using a set of domain parameters, a private key  $x$ , a per-message secret number  $k$ , data to be signed, and a hash function. A digital signature is verified using the same domain parameters, a public key  $y$  that is mathematically associated with the private key  $x$  used to generate the digital signature, data to be verified, and the same hash function that was used during signature generation. These parameters are defined as follows:

- $p$  a prime modulus, where  $2^{L-1} < p < 2^L$ , and  $L$  is the bit length of  $p$ . Values for  $L$  are provided in Section 4.2.
- $q$  a prime divisor of  $(p - 1)$ , where  $2^{N-1} < q < 2^N$ , and  $N$  is the bit length of  $q$ . Values for  $N$  are provided in Section 4.2.
- $g$  a generator of a subgroup of order  $q$  in the multiplicative group of  $GF(p)$ , such that  $1 < g < p$ .
- $x$  the private key that must remain secret;  $x$  is a randomly or pseudorandomly generated integer, such that  $0 < x < q$ , i.e.,  $x$  is in the range  $[1, q-1]$ .
- $y$  the public key, where  $y = g^x \bmod p$ .
- $k$  a secret number that is unique to each message;  $k$  is a randomly or pseudorandomly generated integer, such that  $0 < k < q$ , i.e.,  $k$  is in the range  $[1, q-1]$ .

### 4.2 Selection of Parameter Sizes and Hash Functions for DSA

This Standard specifies the following choices for the pair  $L$  and  $N$  (the bit lengths of  $p$  and  $q$ , respectively):

$$L = 1024, N = 160$$

$$L = 2048, N = 224$$

$$L = 2048, N = 256$$

$$L = 3072, N = 256$$

Federal Government entities **shall** generate digital signatures using use one or more of these choices.

An approved hash function, as specified in FIPS 180, **shall** be used during the generation of digital signatures. The security strength associated with the DSA digital signature process is no greater than the minimum of the security strength of the  $(L, N)$  pair and the security strength of the hash function that is employed. Both the security strength of the hash function used and the security strength of the  $(L, N)$  pair **shall** meet or exceed the security strength required for the

digital signature process. The security strength for each  $(L, N)$  pair and hash function is provided in SP 800-57.

SP 800-57 provides information about the selection of the appropriate  $(L, N)$  pair in accordance with a desired security strength for a given time period for the generation of digital signatures. An  $(L, N)$  pair **shall** be chosen that protects the signed information during the entire expected lifetime of that information. For example, if a digital signature is generated in 2009 for information that needs to be protected for five years, and a particular  $(L, N)$  pair is invalid after 2010, then a larger  $(L, N)$  pair **shall** be used that remains valid for the entire period of time that the information needs to be protected.

It is recommended that the security strength of the  $(L, N)$  pair and the security strength of the hash function used for the generation of digital signatures be the same unless an agreement has been made between participating entities to use a stronger hash function. When the length of the output of the hash function is greater than  $N$  (i.e., the bit length of  $q$ ), then the leftmost  $N$  bits of the hash function output block **shall** be used in any calculation using the hash function output during the generation or verification of a digital signature. A hash function that provides a lower security strength than the  $(L, N)$  pair ordinarily **should not** be used, since this would reduce the security strength of the digital signature process to a level no greater than that provided by the hash function.

A Federal Government entity other than a Certification Authority (CA) **should** use only the first three  $(L, N)$  pairs (i.e., the (1024, 160), (2048, 224) and (2048, 256) pairs). A CA **shall** use an  $(L, N)$  pair that is equal to or greater than the  $(L, N)$  pairs used by its subscribers. For example, if subscribers are using the (2048, 224) pair, then the CA **shall** use either the (2048, 224), (2048, 256) or (3072, 256) pair. Possible exceptions to this rule include cross certification between CAs, certifying keys for purposes other than digital signatures and transitioning from one key size or algorithm to another. See SP 800-57 for further guidance.

#### 4.3 DSA Domain Parameters

DSA requires that the private/public key pairs used for digital signature generation and verification be generated with respect to a particular set of domain parameters. These domain parameters may be common to a group of users and may be public. A user of a set of domain parameters (i.e., both the signatory and the verifier) **shall** have assurance of their validity prior to using them (see Section 3). Although domain parameters may be public information, they **shall** be managed so that the correct correspondence between a given key pair and its set of domain parameters is maintained for all parties that use the key pair. A set of domain parameters may remain fixed for an extended time period.

The domain parameters for DSA are the integers  $p$ ,  $q$ , and  $g$ , and optionally, the *domain\_parameter\_seed* and *counter* that were used to generate  $p$  and  $q$  (i.e., the full set of domain parameters is  $(p, q, g \{, \text{domain\_parameter\_seed}, \text{counter}\})$ ).

### 4.3.1 Domain Parameter Generation

Domain parameters may be generated by a trusted third party (a TTP, such as a CA) or by an entity other than a TTP. Assurance of domain parameter validity **shall** be obtained prior to key pair generation, digital signature generation or digital signature verification (see Section 3).

The integers  $p$  and  $q$  **shall** be generated as specified in Appendix A.1. The input to the generation process is the selected values of  $L$  and  $N$  (see Section 4.2); the output of the generation process is the values for  $p$  and  $q$ , and optionally, the values of the *domain\_parameter\_seed* and *counter*.

The generator  $g$  **shall** be generated as specified in Appendix A.2.

The security strength of a hash function used during the generation of the domain parameters **shall** meet or exceed the security strength associated with the  $(L, N)$  pair. Note that this is more restrictive than the hash function that can be used for the digital signature process (see Section 4.2).

### 4.3.2 Domain Parameter Management

Each digital signature key pair **shall** be correctly associated with one specific set of domain parameters (e.g., by a public key certificate that identifies the domain parameters associated with the public key). The domain parameters **shall** be protected from unauthorized modification until the set is deactivated (if and when the set is no longer needed). The same domain parameters may be used for more than one purpose (e.g., the same domain parameters may be used for both digital signatures and key establishment). However, using different values for the generator  $g$  reduces the risk that key pairs generated for one purpose could be accidentally used (successfully) for another purpose.

## 4.4 Key Pairs

Each signatory has a key pair: a private key  $x$  and a public key  $y$  that are mathematically related to each other. The private key **shall** be used for only a fixed period of time (i.e., the private key cryptoperiod) in which digital signatures may be generated; the public key may continue to be used as long as digital signatures that were generated using the associated private key need to be verified (i.e., the public key may continue to be used beyond the cryptoperiod of the associated private key). See SP 800-57 for further guidance.

### 4.4.1 DSA Key Pair Generation

A digital signature key pair  $x$  and  $y$  is generated for a set of domain parameters  $(p, q, g \{, domain\_parameter\_seed, counter\})$ . Methods for the generation of  $x$  and  $y$  are provided in Appendix B.1.

#### 4.4.2 Key Pair Management

Guidance on the protection of key pairs is provided in SP 800-57. The secure use of digital signatures depends on the management of an entity's digital signature key pair as follows:

1. The validity of the domain parameters **shall** be assured prior to the generation of the key pair, or the verification and validation of a digital signature (see Section 3).
2. Each key pair **shall** be associated with the domain parameters under which the key pair was generated.
3. A key pair **shall** only be used to generate and verify signatures using the domain parameters associated with that key pair.
4. The private key **shall** be used only for signature generation as specified in this Standard and **shall** be kept secret; the public key **shall** be used only for signature verification and may be made public.
5. An intended signatory **shall** have assurance of possession of the private key prior to or concurrently with using it to generate a digital signature (see Section 3.1).
6. A private key **shall** be protected from unauthorized access, disclosure and modification.
7. A public key **shall** be protected from unauthorized modification (including substitution). For example, public key certificates that are signed by a CA may provide such protection.
8. A verifier **shall** be assured of a binding between the public key, its associated domain parameters and the key pair owner (see Section 3).
9. A verifier **shall** obtain public keys in a trusted manner (e.g., from a certificate signed by a CA that the entity trusts, or directly from the intended or claimed signatory, provided that the entity is trusted by the verifier and can be authenticated as the source of the signed information that is to be verified).
10. Verifiers **shall** be assured that the claimed signatory is the key pair owner, and that the owner possessed the private key that was used to generate the digital signature at the time that the signature was generated (i.e., the private key that is associated with the public key that will be used to verify the digital signature) (see Section 3.3).
11. A signatory and a verifier **shall** have assurance of the validity of the public key (see Sections 3.1 and 3.3).

#### 4.5 DSA Per-Message Secret Number

A new secret random number  $k$  **shall** be generated prior to the generation of each digital signature for use during the signature generation process. This secret number **shall** be protected from unauthorized disclosure and modification.

$k^{-1}$  is the multiplicative inverse of  $k$  with respect to multiplication modulo  $q$ ; i.e.,  $0 < k^{-1} < q$

and  $1 = (k^{-1} k) \bmod q$ . This inverse is required for the signature generation process (see Section 4.6). A technique is provided in Appendix C.1 for deriving  $k^{-1}$  from  $k$ .

$k$  and  $k^{-1}$  may be pre-computed, since knowledge of the message to be signed is not required for the computations. When  $k$  and  $k^{-1}$  are pre-computed, their confidentiality and integrity **shall** be protected.

Methods for the generation of the per-message secret number are provided in Appendix B.2.

#### 4.6 DSA Signature Generation

The intended signatory **shall** have assurances as specified in Section 3.1.

Let  $N$  be the bit length of  $q$ . Let  $\min(N, outlen)$  denote the minimum of the positive integers  $N$  and  $outlen$ , where  $outlen$  is the bit length of the hash function output block.

The signature of a message  $M$  consists of the pair of numbers  $r$  and  $s$  that is computed according to the following equations:

$$r = (g^k \bmod p) \bmod q.$$

$$z = \text{the leftmost } \min(N, outlen) \text{ bits of } \mathbf{Hash}(M).$$

$$s = (k^{-1}(z + xr)) \bmod q.$$

When computing  $s$ , the string  $z$  obtained from  $\mathbf{Hash}(M)$  **shall** be converted to an integer. The conversion rule is provided in Appendix C.2.

Note that  $r$  may be computed whenever  $k, p, q$  and  $g$  are available, e.g., whenever the domain parameters  $p, q$  and  $g$  are known, and  $k$  has been pre-computed (see Section 4.5),  $r$  may also be pre-computed, since knowledge of the message to be signed is not required for the computation of  $r$ . Pre-computed  $k, k^{-1}$  and  $r$  values **shall** be protected in the same manner as the the private key  $x$  until  $s$  has been computed (see SP 800-57).

The values of  $r$  and  $s$  **shall** be checked to determine if  $r = 0$  or  $s = 0$ . If either  $r = 0$  or  $s = 0$ , a new value of  $k$  **shall** be generated, and the signature **shall** be recalculated. It is extremely unlikely that  $r = 0$  or  $s = 0$  if signatures are generated properly.

The signature  $(r, s)$  may be transmitted along with the message to the verifier.

#### 4.7 DSA Signature Verification and Validation

Signature verification may be performed by any party (i.e., the signatory, the intended recipient or any other party) using the signatory's public key. A signatory may wish to verify that the computed signature is correct, perhaps before sending the signed message to the intended

recipient. The intended recipient (or any other party) verifies the signature to determine its authenticity.

Prior to verifying the signature of a signed message, the domain parameters, and the claimed signatory's public key and identity **shall** be made available to the verifier in an authenticated manner. The public key may, for example, be obtained in the form of a certificate signed by a trusted entity (e.g., a CA) or in a face-to-face meeting with the public key owner.

Let  $M'$ ,  $r'$ , and  $s'$  be the received versions of  $M$ ,  $r$ , and  $s$ , respectively; let  $y$  be the public key of the claimed signatory; and let  $N$  be the bit length of  $q$ . Also, let  $\min(N, \text{outlen})$  denote the minimum of the positive integers  $N$  and  $\text{outlen}$ , where  $\text{outlen}$  is the bit length of the hash function output block.

The signature verification process is as follows:

1. The verifier **shall** check that  $0 < r' < q$  and  $0 < s' < q$ ; if either condition is violated, the signature **shall** be rejected as invalid.
2. If the two conditions in step 1 are satisfied, the verifier computes the following:

$$w = (s')^{-1} \bmod q.$$

$$z = \text{the leftmost } \min(N, \text{outlen}) \text{ bits of } \mathbf{Hash}(M').$$

$$u1 = (zw) \bmod q.$$

$$u2 = ((r')w) \bmod q.$$

$$v = (((g)^{u1} (y)^{u2}) \bmod p) \bmod q.$$

A technique is provided in Appendix C.1 for deriving  $(s')^{-1}$  (i.e., the multiplicative inverse of  $s' \bmod q$ ).

The string  $z$  obtained from  $\mathbf{Hash}(M')$  **shall** be converted to an integer. The conversion rule is provided in Appendix C.2.

3. If  $v = r'$ , then the signature is verified. For a proof that  $v = r'$  when  $M' = M$ ,  $r' = r$ , and  $s' = s$ , see Appendix E.
4. If  $v$  does not equal  $r'$ , then the message or the signature may have been modified, there may have been an error in the signatory's generation process, or an imposter (who did not know the private key associated with the public key of the claimed signatory) may have attempted to forge the signature. The signature **shall** be considered invalid. No inference can be made as to whether the data is valid, only that when using the public key to verify the signature, the signature is incorrect for that data.
5. Prior to accepting the signature as valid, the verifier **shall** have assurances as specified in Section 3.3.

An organization's policy may govern the action to be taken for invalid digital signatures. Such policy is outside the scope of this Standard. Guidance about determining the timeliness of digitally signed messages is addressed in SP 800-102, Recommendation for Digital Signature Timeliness.

## 5. The RSA Digital Signature Algorithm

The use of the RSA algorithm for digital signature generation and verification is specified in American National Standard (ANS) X9.31 and Public Key Cryptography Standard (PKCS) #1. While each of these standards uses the RSA algorithm, the format of the ANS X9.31 and PKCS #1 data on which the digital signature is generated differs in details that make the algorithms non-interchangeable.

### 5.1 RSA Key Pair Generation

An RSA digital signature key pair consists of an RSA private key, which is used to compute a digital signature, and an RSA public key, which is used to verify a digital signature. An RSA key pair used for digital signatures **shall** only be used for one digital signature scheme (e.g., ANS X9.31, RSASSA-PKCS1 v1.5 or RSASSA-PSS; see Sections 5.4 and 5.5). In addition, an RSA digital signature key pair **shall not** be used for other purposes (e.g., key establishment).

An RSA public key consists of a modulus  $n$ , which is the product of two positive prime integers  $p$  and  $q$  (i.e.,  $n = pq$ ), and a public key exponent  $e$ . Thus, the RSA public key is the pair of values  $(n, e)$  and is used to verify digital signatures. The size of an RSA key pair is commonly considered to be the length of the modulus  $n$  in bits ( $nlen$ ).

The corresponding RSA private key consists of the same modulus  $n$  and a private key exponent  $d$  that depends on  $n$  and the public key exponent  $e$ . Thus, the RSA private key is the pair of values  $(n, d)$  and is used to generate digital signatures. Note that an alternative method for representing  $(n, d)$  using the Chinese Remainder Theorem (CRT) is allowed.

In order to provide security for the digital signature process, the two integers  $p$  and  $q$ , and the private key exponent  $d$  **shall** be kept secret. The modulus  $n$  and the public key exponent  $e$  may be made known to anyone. Guidance on the protection of these values is provided in SP 800-57.

This Standard specifies three choices for the length of the modulus (i.e.,  $nlen$ ): 1024, 2048 and 3072 bits. Federal Government entities **shall** generate digital signatures using one or more of these choices.

An approved hash function, as specified in FIPS 180, **shall** be used during the generation of key pairs and digital signatures. When used during the generation of an RSA key pair (as specified in this Standard), the length in bits of the hash function output block **shall** meet or exceed the security strength associated with the bit length of the modulus  $n$  (see SP 800-57).

The security strength associated with the RSA digital signature process is no greater than the minimum of the security strength associated with the bit length of the modulus and the security strength of the hash function that is employed. The security strength for each modulus length and hash function used during the digital signature process is provided in SP 800-57. Both the security strength of the hash function used and the security strength associated with the bit length



of the modulus  $n$  **shall** meet or exceed the security strength required for the digital signature process.

It is recommended that the security strength of the modulus and the security strength of the hash function be the same unless an agreement has been made between participating entities to use a stronger hash function. A hash function that provides a lower security strength than the security strength associated with the bit length of the modulus ordinarily **should not** be used, since this would reduce the security strength of the digital signature process to a level no greater than that provided by the hash function.

Federal Government entities other than CAs **should** use only the first two choices (i.e.,  $nlen = 1024$  or  $2048$ ) during the timeframes indicated in SP 800-57. A CA **should** use a modulus whose length  $nlen$  is equal to or greater than the moduli used by its subscribers. For example, if the subscribers are using  $nlen = 2048$ , then the CA **should** use  $nlen \geq 2048$ . SP 800-57 provides further information about the selection of the bit length of  $n$ . Possible exceptions to this rule include cross certification between CAs, certifying keys for purposes other than digital signatures and transitioning from one key size or algorithm to another.

Criteria for the generation of RSA key pairs are provided in Appendix B.3.1.

When RSA parameters are randomly generated (i.e., the primes  $p$  and  $q$ , and optionally, the public key exponent  $e$ ), they **shall** be generated using an approved random bit generator. The (pseudo) random bits produced by the random bit generator **shall** be used as seeds for generating RSA parameters (e.g., the (pseudo) random number is used as a prime number generation seed). Prime number generation seeds **shall** be kept secret or destroyed when the modulus  $n$  is computed. If any prime number generation seed is retained (e.g., to regenerate the RSA modulus  $n$ , or as evidence that the generated prime factors  $p$  and  $q$  were generated in compliance with this Standard, then the seed **shall** be kept secret and **shall** be protected. The strength of this protection **shall** be (at least) equivalent to the protection required for the associated private key.

## 5.2 Key Pair Management

The secure use of digital signatures depends on the management of an entity's digital signature key pair. Key pair management requirements for RSA are provided in Section 4.4.2, requirements 4 – 11. Note that the first three requirements in Section 4.4.2, which address the relationship between domain parameters and key pairs, are not applicable to RSA.

## 5.3 Assurances

The intended signatory **shall** have assurances as specified in Section 3.1. Prior to accepting a digital signature as valid, the verifier **shall** have assurances as specified in Section 3.3.

## 5.4 ANS X9.31

ANS X9.31, *Digital Signatures Using Reversible Public Key Cryptography for the Financial Services Industry (rDSA)*, was developed for the American National Standards Institute by the Accredited Standards Committee on Financial Services, X9. See <http://www.x9.org> for information about obtaining copies of ANS X9.31 and any associated errata. The following discussions are based on the version of ANS X9.31 that was approved in 1998.

Methods for the generation of the private prime factors  $p$  and  $q$  are provided in Appendix B.3.

In ANS X9.31, the length of the modulus  $n$  is allowed in increments of 256 bits beyond a minimum of 1024 bits. Implementations claiming conformance to FIPS 186-4 **shall** include one or more of the modulus sizes specified in Section 5.1.

Two methods for the generation of digital signatures are included in ANS X9.31. When the public signature verification exponent  $e$  is odd, the digital signature algorithm is commonly known as RSA; when the public signature verification exponent  $e$  is even, the digital signature algorithm is commonly known as Rabin-Williams. This Standard (i.e., FIPS 186-4) adopts the use of RSA, but does not adopt the use of Rabin-Williams.

During signature verification, the extraction of the hash value  $H(M)'$  from the data structure  $IR'$  **shall** be accomplished by either:

- Selecting the *hashlen* bytes of the data structure  $IR'$  that immediately precedes the two bytes of trailer information, where *hashlen* is the length in bytes of the hash function used, regardless of the length of the padding, or
- If the hash value  $H(M)'$  is selected by its location with respect to the last byte of padding (i.e., 0xBA), including a check that the hash value is followed by only two bytes containing the expected trailer value.

ANS X9.31 contains an annex on random number generation. However, implementations of ANS X9.31 **shall** use approved random number generation methods.

Annexes in ANS X9.31 provide informative discussions of security and implementation considerations.

## 5.5 PKCS #1

Public-Key Cryptography Standard (PKCS) #1, *RSA Cryptography Standard*, defines mechanisms for encrypting and signing data using the RSA algorithm. PKCS #1 v2.1 specifies two digital signature processes and corresponding formats: RSASSA-PKCS1-v1.5 and RSASSA-PSS. Both signature schemes are approved for use, but additional constraints are imposed beyond those specified in PKCS #1 v2.1.

- (a) Implementations that generate RSA key pairs **shall** use the criteria and methods in Appendix B.3 to generate those key pairs,

- (b) Only approved hash functions **shall** be used.
- (c) Only two prime factors  $p$  and  $q$  **shall** be used to form the modulus  $n$ .
- (d) Random numbers **shall** be generated using an **approved** random bit generator.
- (e) For RSASSA-PSS:
  - If  $nlen = 1024$  bits (i.e., 128 bytes), and the output length of the **approved** hash function output block is 512 bits (i.e., 64 bytes), then the length (in bytes) of the salt ( $sLen$ ) **shall** satisfy  $0 \leq sLen \leq hLen - 2$ ,
  - Otherwise, the length (in bytes) of the salt ( $sLen$ ) **shall** satisfy  $0 \leq sLen \leq hLen$ ,  
where  $hLen$  is the length of the hash function output block (in bytes).
- (f) For RSASSA-PKCS-v1.5, when the hash value is recovered from the encoded message  $EM$  during the verification of the digital signature<sup>1</sup>, the extraction of the hash value **shall** be accomplished by either:
  - Selecting the rightmost (least significant) bits of  $EM$ , based on the size of the hash function used, regardless of the length of the padding, or
  - If the hash value is selected by its location with respect to the last byte of padding, including a check that the hash value is located in the rightmost (least significant) bytes of  $EM$  (i.e., no other information follows the hash value in the encoded message).

Note: PKCS #1 was initially developed by RSA Laboratories in 1991 and has been revised as multiple versions. At the time of the approval of FIPS 186-4, three versions of PKCS #1 were available: version 1.5, version 2.0 and version 2.1. This Standard references only version 2.1.

---

<sup>1</sup> PKCS #1, v2.1 provides two methods for comparing the hash values: by comparing the encoded messages  $EM$  and  $EM'$ , and by extracting the hash value from the decoding of the encoded message (see the Note in PKCS #1, v2.1). Step (f) above applies to the latter case.

## 6. The Elliptic Curve Digital Signature Algorithm (ECDSA)

ANS X9.62, *Public Key Cryptography for the Financial Services Industry: The Elliptic Curve Digital Signature Standard (ECDSA)*, was developed for the American National Standards Institute by the Accredited Standards Committee on Financial Services, X9. Information about obtaining copies of ANS X9.62 is available at <http://www.x9.org>. The following discussions are based on the version of ANS X9.62 that was approved in 2005. This version of ANS X9.62 **shall** be used, subject to the transition period referenced in the implementation schedule of this Standard.

ANS X9.62 defines methods for digital signature generation and verification using the Elliptic Curve Digital Signature Algorithm (ECDSA). Specifications for the generation of the domain parameters used during the generation and verification of digital signatures are also included in ANS X9.62. ECDSA is the elliptic curve analog of DSA. ECDSA keys **shall not** be used for any other purpose (e.g., key establishment).

### 6.1 ECDSA Domain Parameters

ECDSA requires that the private/public key pairs used for digital signature generation and verification be generated with respect to a particular set of domain parameters. These domain parameters may be common to a group of users and may be public. Domain parameters may remain fixed for an extended time period.

Domain parameters for ECDSA are of the form  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$ , where  $q$  is the field size;  $FR$  is an indication of the basis used;  $a$  and  $b$  are two field elements that define the equation of the curve;  $domain\_parameter\_seed$  is the domain parameter seed and is an optional bit string that is present if the elliptic curve was randomly generated in a verifiable fashion,  $G$  is a base point of prime order on the curve (i.e.,  $G = (x_G, y_G)$ ),  $n$  is the order of the point  $G$ , and  $h$  is the cofactor (which is equal to the order of the curve divided by  $n$ ).

#### 6.1.1 Domain Parameter Generation

This Standard specifies five ranges for  $n$  (see Table 1). For each range, a maximum cofactor size is also specified. Note that the specification of a cofactor  $h$  in a set of domain parameters is optional in ANS X9.62, whereas implementations conforming to this Standard (i.e., FIPS 186-4) **shall** specify the cofactor  $h$  in the set of domain parameters. Table 1 provides the maximum sizes for the cofactor  $h$ .

**Table 1: ECDSA Security Parameters**

Bit length of $n$	Maximum Cofactor ( $h$ )
160 - 223	$2^{10}$
224 - 255	$2^{14}$
256 - 383	$2^{16}$
384 - 511	$2^{24}$
$\geq 512$	$2^{32}$

ECDSA is defined for two arithmetic fields: the finite field  $GF_p$  and the finite field  $GF_{2^m}$ . For the field  $GF_p$ ,  $p$  is required to be an odd prime.

NIST Recommended curves are provided in Appendix D of this Standard (i.e., FIPS 186-4). Three types of curves are provided:

1. Curves over prime fields, which are identified as P-xxx,
2. Curves over Binary fields, which are identified as B-xxx, and
3. Koblitz curves, which are identified as K-xxx,

where xxx indicates the bit length of the field size.

Alternatively, ECDSA domain parameters may be generated as specified in ANS X9.62; when ECDSA domain parameters are generated (i.e., the NIST Recommended curves are not used), the value of  $G$  **should** be generated canonically (verifiably random). An approved hash function, as specified in FIPS 180, **shall** be used during the generation of ECDSA domain parameters. When generating these domain parameters, the security strength of a hash function used **shall** meet or exceed the security strength associated with the bit length of  $n$  (see footnote 2 below).

An approved hash function, as specified in FIPS 180, is required during the generation of domain parameters. The security strength of the hash function used **shall** meet or exceed the security strength associated with the bit length of  $n$ . The security strengths for the ranges of  $n$  and the hash functions are provided in SP 800-57. It is recommended that the security strength associated with the bit length of  $n$  and the security strength of the hash function be the same

---

<sup>2</sup> The NIST Recommended curves were generated prior to the formulation of this guidance and using SHA-1, which was the only approved hash function available at that time. Since SHA-1 was considered secure at the time of generation, the curves were made public, and SHA-1 will only be used to validate those curves, the NIST Recommended curves are still considered secure and appropriate for Federal government use.

unless an agreement has been made between participating entities to use a stronger hash function; a hash function that provides a lower security strength than is associated with the bit length of  $n$  **shall not** be used. If the length of the output of the hash function is greater than the bit length of  $n$ , then the leftmost  $n$  bits of the hash function output block **shall** be used in any calculation using the hash function output during the generation or verification of a digital signature.

Normally, a CA **should** use a bit length of  $n$  whose assessed security strength is equal to or greater than the assessed security strength associated with the bit length of  $n$  used by its subscribers. For example, if subscribers are using a bit length of  $n$  with an assessed security strength of 112 bits, then CAs **should** use a bit length of  $n$  whose assessed security strength is equal to or greater than 112 bits. SP 800-57 provides further information about the selection of a bit length of  $n$ . Possible exceptions to this rule include cross certification between CAs, certifying keys for purposes other than digital signatures and transitioning from one key size or algorithm to another. However, these exceptions require further analysis.

### 6.1.2 Domain Parameter Management

Each ECDSA key pair **shall** be correctly associated with one specific set of domain parameters (e.g., by a public key certificate that identifies the domain parameters associated with the public key). The domain parameters **shall** be protected from unauthorized modification until the set is deactivated (if and when the set is no longer needed). The same domain parameters may be used for more than one purpose (e.g., the same domain parameters may be used for both digital signatures and key establishment). However, using different domain parameters reduces the risk that key pairs generated for one purpose could be accidentally used (successfully) for another purpose.

## 6.2 Private/Public Keys

An ECDSA key pair consists of a private key  $d$  and a public key  $Q$  that is associated with a specific set of ECDSA domain parameters;  $d$ ,  $Q$  and the domain parameters are mathematically related to each other. The private key is normally used for a period of time (i.e., the cryptoperiod); the public key may continue to be used as long as digital signatures that have been generated using the associated private key need to be verified (i.e., the public key may continue to be used beyond the cryptoperiod of the associated private key). See SP 800-57 for further guidance.

ECDSA keys **shall** only be used for the generation and verification of ECDSA digital signatures.

### 6.2.1 Key Pair Generation

A digital signature key pair  $d$  and  $Q$  is generated for a set of domain parameters  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$ . Methods for the generation of  $d$  and  $Q$  are provided in Appendix B.4.

### 6.2.2 Key Pair Management

The secure use of digital signatures depends on the management of an entity's digital signature key pair as specified in Section 4.4.2.

### 6.3 Secret Number Generation

A new secret random number  $k$  **shall** be generated prior to the generation of each digital signature for use during the signature generation process. This secret number **shall** be protected from unauthorized disclosure and modification. Methods for the generation of the per-message secret number are provided in Appendix B.5.

$k^{-1}$  is the multiplicative inverse of  $k$  with respect to multiplication modulo  $n$ ; i.e.,  $0 < k^{-1} < n$  and  $1 = (k^{-1} k) \bmod n$ . This inverse is required for the signature generation process. A technique is provided in Appendix C.1 for deriving  $k^{-1}$  from  $k$ .

$k$  and  $k^{-1}$  may be pre-computed, since knowledge of the message to be signed is not required for the computations. When  $k$  and  $k^{-1}$  are pre-computed, their confidentiality and integrity **shall** be protected.

### 6.4 ECDSA Digital Signature Generation and Verification

An ECDSA digital signature  $(r, s)$  **shall** be generated as specified in ANS X9.62, using:

1. Domain parameters that are generated in accordance with Section 6.1.1,
2. A private key that is generated as specified in Section 6.2.1,
3. A per-message secret number that is generated as specified in Section 6.3,
4. An approved hash function as discussed below, and
5. An approved random bit generator.

An ECDSA digital signature **shall** be verified as specified in ANS X9.62, using the same domain parameters and hash function that were used during signature generation.

An approved hash function, as specified in FIPS 180, **shall** be used during the generation of digital signatures. The security strength associated with the ECDSA digital signature process is no greater than the minimum of the security strength associated with the bit length of  $n$  and the security strength of the hash function that is employed. Both the security strength of the hash

function used and the security strength associated with the bit length of  $n$  **shall** meet or exceed the security strength required for the digital signature process. The security strengths for the ranges of the bit lengths of  $n$  and for each hash function is provided in SP 800-57.

It is recommended that the security strength associated with the bit length of  $n$  and the security strength of the hash function be the same unless an agreement has been made between participating entities to use a stronger hash function. When the length of the output of the hash function is greater than the bit length of  $n$ , then the leftmost  $n$  bits of the hash function output block **shall** be used in any calculation using the hash function output during the generation or verification of a digital signature. A hash function that provides a lower security strength than the security strength associated with the bit length of  $n$  ordinarily **should not** be used, since this would reduce the security strength of the digital signature process to a level no greater than that provided by the hash function.

## **6.5 Assurances**

The intended signatory **shall** have assurances as specified in Section 3.1. Prior to accepting a signature as valid, the verifier **shall** have assurances as specified in Section 3.3.



## APPENDIX A: Generation and Validation of FFC Domain Parameters

Finite field cryptography (FFC) is a method of implementing discrete logarithm cryptography using finite field mathematics. DSA, as specified in this Standard, is an example of FFC. The Diffie-Hellman and MQV key establishment algorithms specified in SP 800-56A can also be implemented as FFC.

The domain parameters for FFC consist of the set of values  $(p, q, g \{, domain\_parameter\_seed, counter\})$ . This appendix specifies techniques for the generation of the FFC domain parameters  $p, q$  and  $g$  and performing an explicit domain parameter validation. During the generation process, the values for *domain\_parameter\_seed* and *counter* are obtained.

### A.1 Generation of the FFC Primes $p$ and $q$

This section provides methods for generating the primes  $p$  and  $q$  that fulfill the criteria specified in Sections 4.1 and 4.2. One of these methods **shall** be used when generating these primes. A method is provided in Appendix A.1.1 to generate random candidate integers and then test them for primality using a probabilistic algorithm. A second method is provided in Appendix A.1.2 that constructs integers from smaller integers so that the constructed integer is guaranteed to be prime.

During the generation, validation and testing processes, conversions between bit strings and integers are required. Appendix C.2 provides methods for these conversions.

#### A.1.1 Generation and Validation of Probable Primes

Previous versions of this Standard contained a method for the generation of the domain parameters  $p$  and  $q$  using SHA-1 and probabilistic methods. This method is no longer approved for domain parameter generation; however, the validation process for this method is provided in Appendix A.1.1.1 to validate previously generated domain parameters.

A method for the generation and validation of the primes  $p$  and  $q$  using probabilistic methods is provided in Appendix A.1.1.2 and is based on the use of an approved hash function; this method **shall** be used for generating probable primes. The validation process for this method is provided in Appendix A.1.1.3.

The probabilistic methods use a hash function and an arbitrary seed (*domain\_parameter\_seed*). Arbitrary seeds could be anything, e.g., a user's favorite number or a random or pseudorandom number output by an approved random number generator. The *domain\_parameter\_seed* determines a sequence of candidates for  $p$  and  $q$  in the required intervals that are then tested for primality using a probabilistic primality test (see Appendix C.3). The test determines that the candidate is either not a prime (i.e., it is a composite integer) or is "probably a prime" (i.e., there is a very small probability that a composite integer will be declared to be a prime).  $p$  and  $q$  **shall** be the first candidate set that passes the primality tests. Note that the *domain\_parameter\_seed*

**shall** be unique for every unique set of domain parameters that are generated using the same method.

#### **A.1.1.1 Validation of the Probable Primes $p$ and $q$ that were Generated Using SHA-1 as Specified in Prior Versions of this Standard**

This prime validation algorithm is used to validate that the primes  $p$  and  $q$  that were generated by the prime generation algorithm specified in previous versions of this Standard. The algorithm requires the values of  $p$ ,  $q$ , *domain\_parameter\_seed* and *counter*, which were output from the prime generation algorithm.

Let **SHA1**( ) be the SHA-1 hash function specified in FIPS 180. The following process or its equivalent **shall** be used to validate  $p$  and  $q$  for this method.

##### **Input:**

1.  $p, q$  The generated primes  $p$  and  $q$ .
2. *domain\_parameter\_seed* A seed that was used to generate  $p$  and  $q$ .
3. *counter* A count value that was determined during generation.

##### **Output:**

1. *status* The status returned from the validation procedure, where status is either **VALID** or **INVALID**.

##### **Process:**

1. If (**len** ( $p$ )  $\neq$  1024) or (**len** ( $q$ )  $\neq$  160), then return **INVALID**.
2. If (*counter* > 4095), then return **INVALID**.
3. *seedlen* = **len** (*domain\_parameter\_seed*).
4. If (*seedlen* < 160), then return **INVALID**.
5. *computed\_q* = **SHA1**(*domain\_parameter\_seed*)  $\oplus$  **SHA1**((*domain\_parameter\_seed* + 1) mod  $2^{\text{seedlen}}$ ).
6. Set the first and last bits of *computed\_q* equal to 1 (i.e., the 159<sup>th</sup> and 0<sup>th</sup> bits).
7. Test whether or not *computed\_q* is prime as specified in Appendix C.3. If (*computed\_q*  $\neq$   $q$ ) or (*computed\_q* is not prime), then return **INVALID**.
8. *offset* = 2.
9. For  $i = 0$  to *counter* do
  - 9.1 For  $j = 0$  to 6 do
$$V_j = \text{SHA1}((\text{domain\_parameter\_seed} + \text{offset} + j) \bmod 2^{\text{seedlen}}).$$
  - 9.2 
$$W = V_0 + (V_1 * 2^{160}) + (V_2 * 2^{320}) + (V_3 * 2^{480}) + (V_4 * 2^{640}) + (V_5 * 2^{800}) +$$

$$((V_6 \bmod 2^{63}) * 2^{960}).$$

- 9.3  $X = W + 2^{1023}$ . Comment:  $0 \leq W < 2^{L-1}$ .
- 9.4  $c = X \bmod 2q$ .
- 9.5  $computed\_p = X - (c - 1)$ . Comment:  $computed\_p \equiv 1 \pmod{2q}$ .
- 9.6 If  $(computed\_p < 2^{1023})$ , then go to step 9.8.
- 9.7 Test whether or not  $computed\_p$  is prime as specified in Appendix C.3. If  $computed\_p$  is determined to be prime, then go to step 10.
- 9.8  $offset = offset + 7$ .
10. If  $((i \neq counter) \text{ or } (computed\_p \neq p) \text{ or } (computed\_p \text{ is not prime}))$ , then return **INVALID**.
11. Return **VALID**.

#### A.1.1.2 Generation of the Probable Primes $p$ and $q$ Using an Approved Hash Function

This method uses an approved hash function and may be used for the generation of the primes  $p$  and  $q$  for any application (e.g., digital signatures or key establishment). The security strength of the hash function **shall** be equal to or greater than the security strength associated with the  $(L, N)$  pair.

An arbitrary *domain\_parameter\_seed* of *seedlen* bits is also used, where *seedlen* **shall** be equal to or greater than  $N$ .

The generation process returns a set of integers  $p$  and  $q$  that have a very high probability of being prime. For another entity to validate that the primes were generated correctly using the validation process in Appendix A.1.1.3, the value of the *domain\_parameter\_seed* and the *counter* used to generate the primes must also be returned and made available to the validating entity; the *domain\_parameter\_seed* and *counter* need not be kept secret. Let **Hash**( ) be the selected hash function, and let *outlen* be the bit length of the output block, where *outlen* **shall** be equal to or greater than  $N$ .

The following process or its equivalent **shall** be used to generate  $p$  and  $q$  for this method.

##### Input:

1.  $L$  The desired length of the prime  $p$  (in bits).
2.  $N$  The desired length of the prime  $q$  (in bits).
3. *seedlen* The desired length of the domain parameter seed; *seedlen* **shall** be equal to or greater than  $N$ .

##### Output:

1. *status* The status returned from the generation procedure, where status is

either **VALID** or **INVALID**. If **INVALID** is returned as the *status*, either no values for the other output parameters **shall** be returned, or invalid values **shall** be returned (e.g., zeros or Null strings).

2.  $p, q$  The generated primes  $p$  and  $q$ .
3. *domain\_parameter\_seed*  
(Optional) A seed that was used to generate  $p$  and  $q$ .
4. *counter* (Optional) A count value that was determined during generation.

**Process:**

1. Check that the  $(L, N)$  pair is in the list of acceptable  $(L, N)$  pairs (see Section 4.2). If the pair is not in the list, then return **INVALID**.
2. If  $(seedlen < N)$ , then return **INVALID**.
3.  $n = \lceil L/outlen \rceil - 1$ .
4.  $b = L - 1 - (n * outlen)$ .
5. Get an arbitrary sequence of  $seedlen$  bits as the *domain\_parameter\_seed*.
6.  $U = \mathbf{Hash}(\text{domain\_parameter\_seed}) \bmod 2^{N-1}$ .
7.  $q = 2^{N-1} + U + 1 - (U \bmod 2)$ .
8. Test whether or not  $q$  is prime as specified in Appendix C.3.
9. If  $q$  is not a prime, then go to step 5.
10.  $offset = 1$ .
11. For  $counter = 0$  to  $(4L - 1)$  do
  - 11.1 For  $j = 0$  to  $n$  do
$$V_j = \mathbf{Hash}((\text{domain\_parameter\_seed} + offset + j) \bmod 2^{seedlen}).$$
  - 11.2  $W = V_0 + (V_1 * 2^{outlen}) + \dots + (V_{n-1} * 2^{(n-1) * outlen}) + ((V_n \bmod 2^b) * 2^{n * outlen})$ .
  - 11.3  $X = W + 2^{L-1}$ . Comment:  $0 \leq W < 2^{L-1}$ ; hence,  $2^{L-1} \leq X < 2^L$ .
  - 11.4  $c = X \bmod 2q$ .
  - 11.5  $p = X - (c - 1)$ . Comment:  $p \equiv 1 \pmod{2q}$ .
  - 11.6 If  $(p < 2^{L-1})$ , then go to step 11.9.
  - 11.7 Test whether or not  $p$  is prime as specified in Appendix C.3.
  - 11.8 If  $p$  is determined to be prime, then return **VALID** and the values of  $p, q$  and (optionally) the values of *domain\_parameter\_seed* and *counter*.

11.9  $offset = offset + n + 1$ .

Comment: Increment *offset*; then, as part of the loop in step 11, increment *counter*; if  $counter < 4L$ , repeat steps 11.1 through 11.8.

12. Go to step 5.

#### A.1.1.3 Validation of the Probable Primes $p$ and $q$ that were Generated Using an Approved Hash Function

This prime validation algorithm is used to validate that the integers  $p$  and  $q$  were generated by the prime generation algorithm given in Appendix A.1.1.2. The validation algorithm requires the values of  $p$ ,  $q$ , *domain\_parameter\_seed* and *counter*, which were output from the prime generation algorithm. Let **Hash**( ) be the hash function used to generate  $p$  and  $q$ , and let *outlen* be its output block length.

The following process or its equivalent **shall** be used to validate  $p$  and  $q$  for this method.

##### Input:

1.  $p, q$  The generated primes  $p$  and  $q$ .
3. *domain\_parameter\_seed* The domain parameter seed that was used to generate  $p$  and  $q$ .
4. *counter* A count value that was determined during generation.

##### Output:

1. *status* The status returned from the validation procedure, where status is either **VALID** or **INVALID**.

##### Process:

1.  $L = \text{len}(p)$ .
2.  $N = \text{len}(q)$ .
3. Check that the  $(L, N)$  pair is in the list of acceptable  $(L, N)$  pairs (see Section 4.2). If the pair is not in the list, return **INVALID**.
4. If  $(counter > (4L - 1))$ , then return **INVALID**.
5.  $seedlen = \text{len}(\text{domain\_parameter\_seed})$ .
6. If  $(seedlen < N)$ , then return **INVALID**.
7.  $U = \text{Hash}(\text{domain\_parameter\_seed}) \bmod 2^{N-1}$ .
8.  $computed\_q = 2^{N-1} + U + 1 - (U \bmod 2)$ .
9. Test whether or not *computed\_q* is prime as specified in Appendix C.3. If  $(computed\_q \neq q)$  or  $(computed\_q \text{ is not prime})$ , then return **INVALID**.

10.  $n = \lceil L / \text{outlen} \rceil - 1$ .
11.  $b = L - 1 - (n * \text{outlen})$ .
12.  $\text{offset} = 1$ .
13. For  $i = 0$  to *counter* do
  - 13.1 For  $j = 0$  to  $n$  do
 
$$V_j = \mathbf{Hash}((\text{domain\_parameter\_seed} + \text{offset} + j) \bmod 2^{\text{seedlen}}).$$
  - 13.2  $W = V_0 + (V_1 * 2^{\text{outlen}}) + \dots + (V_{n-1} * 2^{(n-1) * \text{outlen}}) + ((V_n \bmod 2^b) * 2^{n * \text{outlen}}).$
  - 13.3  $X = W + 2^{L-1}$ .
  - 13.4  $c = X \bmod 2q$ .
  - 13.5  $\text{computed\_p} = X - (c - 1)$ .
  - 13.6 If  $(\text{computed\_p} < 2^{L-1})$ , then go to step 13.9
  - 13.7 Test whether or not  $\text{computed\_p}$  is prime as specified in Appendix C.3.
  - 13.8 If  $\text{computed\_p}$  is determined to be a prime, then go to step 14.
  - 13.9  $\text{offset} = \text{offset} + n + 1$ .
14. If  $((i \neq \text{counter}) \text{ or } (\text{computed\_p} \neq p) \text{ or } (\text{computed\_p} \text{ is not a prime}))$ , then return **INVALID**.
15. Return **VALID**.

### A.1.2 Construction and Validation of the Provable Primes $p$ and $q$

Primes can be generated so that they are guaranteed to be prime. The following algorithm for generating  $p$  and  $q$  uses an approved hash function and an arbitrary seed (*firstseed*) to construct  $p$  and  $q$  in the required intervals. The security strength of the hash function **shall** be equal to or greater than the security strength associated with the  $(L, N)$  pair.

Arbitrary seeds can be anything, e.g., a user's favorite number or a random or pseudorandom number that is output from a random number generator. Note that the *firstseed* must be unique to produce a unique set of domain parameters. Candidate primes are tested for primality using a deterministic primality test that proves whether or not the candidate is prime. The resulting  $p$  and  $q$  are guaranteed to be primes.

#### A.1.2.1 Construction of the Primes $p$ and $q$ Using the Shawe-Taylor Algorithm

For each set of domain parameters generated, an arbitrary initial seed (*firstseed*) of at least *seedlen* bits **shall** be determined, where *seedlen* **shall** be  $\geq N$ .

The generation process returns a set of integers  $p$  and  $q$  that are guaranteed to be prime. For

another entity to validate that the primes were generated correctly (using the validation process in Appendix A.1.2.2), the value of the *firstseed*, the two computed seeds (*pseed* and *qseed*) and the counters used to generate the primes (*pgen\_counter* and *qgen\_counter*) must be made available to the validating entity; the seeds and the counters need not be kept secret. The domain parameters for DSA are identified in Section 4.3 as ( $p, q, g \{, domain\_parameter\_seed, counter\}$ ). When using the Shawe-Taylor algorithm for generating  $p$  and  $q$ , the *domain\_parameter\_seed* consists of three seed values (*firstseed*, *pseed*, and *qseed*), and the *counter* consists of the pair of counter values (*pgen\_counter* and *qgen\_counter*).

Let **Hash**( ) be the selected hash function (see Appendix A.1.2), and let *outlen* be the bit length of the output block of that hash function.

#### A.1.2.1.1 Get the First Seed

The following process or its equivalent **shall** be used to generate *firstseed* for this constructive method.

##### Input:

1.  $N$  The length of  $q$  in bits.
2. *seedlen* The length of *firstseed*, where  $seedlen \geq N$ .

##### Output:

1. *status* The status returned from the generation procedure, where *status* is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned, then either no *firstseed* value **shall** be provided or an invalid value **shall** be returned.
2. *firstseed* The first seed generated.

##### Process:

1.  $firstseed = 0$ .
2. Check that  $N$  is in the list of acceptable ( $L, N$ ) pairs (see Section 4.2). If not, then return **FAILURE**.
3. If ( $seedlen < N$ ), then return **FAILURE**.
4. While  $firstseed < 2^{N-1}$ .  
Get an arbitrary sequence of *seedlen* bits as *firstseed*.
5. Return **SUCCESS** and the value of *firstseed*.

Note: This routine could be incorporated into the beginning of the constructive prime generation procedure in Appendix A.1.2.1.2. However, this was not done in this specification so that the

validation process in Appendix A.1.2.2 could also call the constructive prime generation procedure and provide the value of *firstseed* as input.

#### A.1.2.1.2 Constructive Prime Generation

The following process or its equivalent **shall** be used to generate  $p$  and  $q$  for this constructive method.

##### Input:

1.  $L$  The requested length for  $p$  (in bits).
2.  $N$  The requested length for  $q$  (in bits).
3. *firstseed* The first seed to be used. This was obtained as specified in Appendix A.1.2.1.1.

##### Output:

1. *status* The status returned from the generation procedure, where *status* is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned, then either no other values **shall** be returned, or invalid values **shall** be returned.
2.  $p, q$  The requested primes.
3. *pseed, qseed* (Optional) Computed seed values that were used to generate  $p$  and  $q$ . The entire *seed* for the generation of  $p$  and  $q$  consists of *firstseed, pseed* and *qseed*.
4. *pgen\_counter, qgen\_counter*  
(Optional) The count values that were determined during generation.

##### Process:

1. Check that the  $(L, N)$  pair is in the list of acceptable  $(L, N)$  pairs (see Section 4.2). If the pair is not in the list, return **FAILURE**.  

Comment: Use the Shawe-Taylor random prime routine in Appendix C.6 to generate random primes.
2. Using  $N$  as the length and *firstseed* as the *input\_seed*, use the random prime generation routine in Appendix C.6 to obtain  $q, qseed$  and *qgen\_counter*. If **FAILURE** is returned, then return **FAILURE**.
3. Using  $\lceil L / 2 + 1 \rceil$  as the *length* and *qseed* as the *input\_seed*, use the random prime routine in Appendix C.6 to obtain  $p_0, pseed$ , and *pgen\_counter*. If **FAILURE** is returned, then return **FAILURE**.



4.  $iterations = \lceil L / outlen \rceil - 1$ .

5.  $old\_counter = pgen\_counter$ .

Comment: Generate a (pseudo) random  $x$  in the interval  $[2^{L-1}, 2^L]$ .

6.  $x = 0$ .

7. For  $i = 0$  to  $iterations$  do

$x = x + (\mathbf{Hash}(pseed + i) * 2^{i * outlen})$ .

8.  $pseed = pseed + iterations + 1$ .

9.  $x = 2^{L-1} + (x \bmod 2^{L-1})$ .

Comment: Generate  $p$ , a candidate for the prime, in the interval  $[2^{L-1}, 2^L]$ .

10.  $t = \lceil x / (2q p_0) \rceil$ .

11. If  $(2tq p_0 + 1) > 2^L$ , then  $t = \lceil 2^{L-1} / (2q p_0) \rceil$ .

12.  $p = 2tq p_0 + 1$ .

13.  $pgen\_counter = pgen\_counter + 1$ .

Comment: Test  $p$  for primality; choose an integer  $a$  in the interval  $[2, p-2]$ .

14.  $a = 0$

15. For  $i = 0$  to  $iterations$  do

$a = a + (\mathbf{Hash}(pseed + i) * 2^{i * outlen})$ .

16.  $pseed = pseed + iterations + 1$ .

17.  $a = 2 + (a \bmod (p-3))$ .

18.  $z = a^{2tq} \bmod p$ .

19. If  $((1 = \mathbf{GCD}(z-1, p)) \text{ and } (1 = z^{p_0} \bmod p))$ , then return **SUCCESS** and the values of  $p$ ,  $q$  and (optionally)  $pseed$ ,  $qseed$ ,  $pgen\_counter$ , and  $qgen\_counter$ .

20. If  $(pgen\_counter > (4L + old\_counter))$ , then **return FAILURE**.

21.  $t = t + 1$ .

22. Go to step 11.

### A.1.2.2 Validation of the DSA Primes $p$ and $q$ that were Constructed Using the Shawe-Taylor Algorithm

The validation of the primes  $p$  and  $q$  that were generated by the method described in Appendix A.1.2.1.2 may be performed if the values of *firstseed*, *pseed*, *qseed*, *pgen\_counter* and *qgen\_counter* were saved and are provided for use in the following algorithm.

The following process or its equivalent **shall** be used to validate  $p$  and  $q$  for this constructive method.

#### Input:

1.  $p, q$  The primes to be validated.
2. *firstseed*, *pseed*, *qseed* Seed values that were used to generate  $p$  and  $q$ .
3. *pgen\_counter*, *qgen\_counter* The count values that were determined during generation.

#### Output:

1. *status* The status returned from the validation procedure, where *status* is either **SUCCESS** or **FAILURE**.

#### Process:

1.  $L = \text{len}(p)$ .
2.  $N = \text{len}(q)$ .
3. Check that the  $(L, N)$  pair is in the list of acceptable  $(L, N)$  pairs (see Section 4.2). If the pair is not in the list, then return **FAILURE**.
4. If  $(\text{firstseed} < 2^{N-1})$ , then return **FAILURE**.
5. If  $(2^N \leq q)$ , then return **FAILURE**.
6. If  $(2^L \leq p)$ , then return **FAILURE**.
7. If  $((p - 1) \bmod q \neq 0)$ , then return **FAILURE**.
8. Using  $L, N$  and *firstseed*, perform the constructive prime generation procedure in Appendix A.1.2.1.2 to obtain *p\_val*, *q\_val*, *pseed\_val*, *qseed\_val*, *pgen\_counter\_val*, and *qgen\_counter\_val*. If **FAILURE** is returned, or if  $(q\_val \neq q)$  or  $(qseed\_val \neq$

$qseed$ ) or ( $qgen\_counter\_val \neq qgen\_counter$ ) or ( $p\_val \neq p$ ) or ( $pseed\_val \neq pseed$ ) or ( $pgen\_counter\_val \neq pgen\_counter$ ), then return **FAILURE**.

9. Return **SUCCESS**.

## A.2 Generation of the Generator $g$

The generator  $g$  depends on the values of  $p$  and  $q$ . Two methods for determining the generator  $g$  are provided; one of these methods **shall** be used. The first method, discussed in Appendix A.2.1, may be used when complete validation of the generator  $g$  is not required; it is recommended that this method be used only when the party generating  $g$  is trusted to not deliberately generate a  $g$  that has a potentially exploitable relationship to another generator  $g'$ . For example, it must be hard to determine an exponential relationship between the generators such that  $g = (g')^x \bmod p$  for a known value of  $x$ . (Note: Read  $(g')^x$  as  $g$  prime to the  $x$ .)

Appendix A.2.2 provides a method for partial validation when the method of generation in Appendix A.2.1 is used. The second method for generating  $g$ , discussed in Appendix A.2.3, **shall** be used when validation of the generator  $g$  is required; the method for the validation of a generator determined using the method in Appendix A.2.3 is provided in Appendix A.2.4.

### A.2.1 Unverifiable Generation of the Generator $g$

This method is used to determine a value for  $g$ , based on the values of  $p$  and  $q$ . It may be used when validation of the generator  $g$  is not required. The correct generation of  $g$  cannot be completely validated (see Appendix A.2.2). Note that this generation method for  $g$  was also specified in previous versions of this Standard.

The following process or its equivalent **shall** be used to generate the generator  $g$  for this method.

#### Input:

1.  $p, q$             The generated primes.

#### Output:

1.  $g$             The requested value of  $g$ .

#### Process:

1.  $e = (p - 1)/q$ .
2. Set  $h =$  any integer satisfying  $1 < h < (p - 1)$ , such that  $h$  differs from any value previously tried. Note that  $h$  could be obtained from a random number generator or from a counter that changes after each use.
3.  $g = h^e \bmod p$ .

4. If  $(g = 1)$ , then go to step 2.
5. Return  $g$ .

### A.2.2 Assurance of the Validity of the Generator $g$

The order of the generator  $g$  that was generated using Appendix A.2.1 can be partially validated by checking the range and order, thereby performing a partial validation of  $g$ .

The following process or its equivalent **shall** be used when partial validation of the generator  $g$  is required:

**Input:**

1.  $p, q, g$       The domain parameters.

**Output:**

1.  $status$       The status returned from the generation routine, where  $status$  is either **PARTIALLY VALID** or **INVALID**.

**Process:**

1. Verify that  $2 \leq g \leq (p-1)$ . If not true, return **INVALID**.
2. If  $(g^q = 1 \bmod p)$ , then return **PARTIALLY VALID**.
3. Return **INVALID**.

The non-existence of a potentially exploitable relationship of  $g$  to another generator  $g'$  (that is known to the entity that generated  $g$ , but may not be known by other entities) cannot be checked. In this sense, the correct generation of  $g$  cannot be completely validated.

### A.2.3 Verifiable Canonical Generation of the Generator $g$

The generation of  $g$  is based on the values of  $p, q$  and *domain\_parameter\_seed* (which are outputs of the generation processes in Appendix A.1). When  $p$  and  $q$  were generated using the method in Appendix A.1.1.2, the *domain\_parameter\_seed* value must have been returned from the generation routine. When  $p$  and  $q$  were generated using the method in Appendix A.1.2.1, the *firstseed*, *pseed*, and *qseed* values must have been returned from the generation routine; in this case,  $domain\_parameter\_seed = firstseed || pseed || qseed$  **shall** be used in the following process.

This method of generating a generator  $g$  can be validated (see Appendix A.2.4).

This generation method supports the generation of multiple values of  $g$  for specific values of  $p$  and  $q$ . The use of different values of  $g$  for the same  $p$  and  $q$  may be used to support key separation; for example, using the  $g$  that is generated with  $index = 1$  for digital signatures and with  $index = 2$  for key establishment.

Let **Hash**( ) be the hash function used to generate  $p$  and  $q$  (see Appendix A.1). The following process or its equivalent **shall** be used to generate the generator  $g$ .

**Input:**

1.  $p, q$  The primes.
2.  $domain\_parameter\_seed$  The seed used during the generation of  $p$  and  $q$ .
3.  $index$  The index to be used for generating  $g$ .  $index$  is a bit string of length 8 that represents an unsigned integer.

**Output:**

1.  $status$  The status returned from the generation routine, where  $status$  is either **VALID** or **INVALID**.
2.  $g$  The value of  $g$  that was generated.

**Process:**

Note:  $count$  is an unsigned 16-bit integer.

Comment: Check that a valid value of the  $index$  has been provided (see above).

1. If ( $index$  is incorrect), then return **INVALID**.
2.  $N = \text{len}(q)$ .
3.  $e = (p - 1)/q$ .
4.  $count = 0$ .
5.  $count = count + 1$ .

Comment: Check that  $count$  does not wrap around to 0.

6. If ( $count = 0$ ), then return **INVALID**.

Comment: the length of the  $domain\_parameter\_seed$  has already been checked. "ggen" is the bit string 0x6767656E.

7.  $U = domain\_parameter\_seed \parallel \text{"ggen"} \parallel index \parallel count$ .
8.  $W = \text{Hash}(U)$ .
9.  $g = W^e \bmod p$ .
10. If ( $g < 2$ ), then go to step 5. Comment: If a generator has not been found.
11. Return **VALID** and the value of  $g$ .

#### A.2.4 Validation Routine when the Canonical Generation of the Generator $g$ Routine Was Used

This algorithm **shall** be used to validate the value of  $g$  that was generated using the process in Appendix A.2.3, based on the values of  $p$ ,  $q$ , *domain\_parameter\_seed*, and the appropriate value of *index*. It is assumed that the values of  $p$  and  $q$  have been previously validated according to Appendix A.1. Note that the method specified in Appendix A.2.3 for the generation of  $g$  was not included in previous versions of this Standard; therefore, this validation method is not appropriate for that case.

The *domain\_parameter\_seed* is an output from the generation of  $p$  and  $q$ . When  $p$  and  $q$  were generated using the method in Appendix A.1.1.2, the *domain\_parameter\_seed* must have been returned from the generation routine and made available to the validating party. When  $p$  and  $q$  were generated using the method in Appendix A.1.2.1, the *firstseed*, *pseed*, and *qseed* values must have been returned from the generation routine and made available; *firstseed*, *pseed*, and *qseed* **shall** be concatenated to form the *domain\_parameter\_seed* used in the following process. Let **Hash**( ) be the hash function used to generate  $g$  (i.e., the hash function also used to generate  $p$  and  $q$ ).

The input *index* is the index number for the generator  $g$ . See Appendix A.2.3 for more details.

The following process or its equivalent **shall** be used to validate the generator  $g$  for this method.

##### Input:

- |                                 |  |
|---------------------------------|--|
| 1. $p, q$                       | The primes.  |
| 2. <i>domain_parameter_seed</i> | The seed used to generate $p$ and $q$ .  |
| 3. <i>index</i>                 | The index used in Appendix A.2.3 to generate $x$ . <i>index</i> is a bit string of length 8 that represents an unsigned integer. |
| 4. $g$                          | The value of $g$ to be validated.  |

##### Output:

- |                  |   |
|------------------|---|
| 1. <i>status</i> | The status returned from the generation routine, where <i>status</i> is either <b>VALID</b> or <b>INVALID</b> . |
|------------------|---|

##### Process:

**Note:** *count* is an unsigned 16-bit integer.

**Comment:** Check that a valid value of the *index* has been provided (see above).

1. If (*index* is incorrect), then return **INVALID**.
2. Verify that  $2 \leq g \leq (p-1)$ . If not true, return **INVALID**.
3. If  $(g^q \neq 1 \bmod p)$ , then return **INVALID**.

4.  $N = \text{len}(q)$ .
5.  $e = (p - 1)/q$ .
6.  $count = 0$ .
7.  $count = count + 1$ .

Comment: Check that *count* does not wrap around to 0.

8. If ( $count = 0$ ), then return **INVALID**.

Comment: “ggen” is the bit string 0x6767656E.

9.  $U = \text{domain\_parameter\_seed} \parallel \text{“ggen”} \parallel \text{index} \parallel \text{count}$ .
10.  $W = \text{Hash}(U)$ .
11.  $\text{computed\_g} = W^e \bmod p$ .
12. If ( $\text{computed\_g} < 2$ ), then go to step 7. Comment: If a generator has not been found.
13. If ( $\text{computed\_g} = g$ ), then return **VALID**, else return **INVALID**.

## APPENDIX B: Key Pair Generation

Discrete logarithm cryptography (DLC) is divided into finite field cryptography (FFC) and elliptic curve cryptography (ECC); the difference between the two is the type of math that is used. DSA is an example of FFC; ECDSA is an example of ECC. Other examples of DLC are the Diffie-Hellman and MQV key agreement algorithms, which have both FFC and ECC forms.

The most common example of integer factorization cryptography (IFC) is RSA.

This appendix specifies methods for the generation of FFC and ECC key pairs and secret numbers, and the generation of IFC key pairs. All generation methods require the use of an approved, properly instantiated random bit generator (RBG); the RBG **shall** have a security strength equal to or greater than the security strength associated with the key pairs and secret numbers to be generated. See SP 800-57 for guidance on security strengths and key sizes.

This appendix does not indicate the required conversions between bit strings and integers. When required by a process in this appendix, the conversion **shall** be accomplished as specified in Appendix C.2.

### B.1 FFC Key Pair Generation

An FFC key pair  $(x, y)$  is generated for a set of domain parameters  $(p, q, g \{, domain\_parameter\_seed, counter\})$ . Two methods are provided for the generation of the FFC private key  $x$  and public key  $y$ ; one of these two methods **shall** be used. Prior to generating DSA key pairs, assurance of the validity of the domain parameters  $(p, q$  and  $g)$  **shall** have been obtained as specified in Section 3.1.

For DSA, the valid values of  $L$  and  $N$  are provided in Section 4.2.

#### B.1.1 Key Pair Generation Using Extra Random Bits

In this method, 64 more bits are requested from the RBG than are needed for  $x$  so that bias produced by the mod function in step 6 is negligible.

The following process or its equivalent may be used to generate an FFC key pair.

**Input:**

$(p, q, g)$	The subset of the domain parameters that are used for this process. $p, q$ and $g$ <b>shall</b> either be provided as integers during input, or <b>shall</b> be converted to integers prior to use.
-------------	---



**Output:**

1. *status* The status returned from the key pair generation process. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(x, y)$  The generated private and public keys. If an error is encountered during the generation process, invalid values for  $x$  and  $y$  **should** be returned, as represented by *Invalid\_x* and *Invalid\_y* in the following specification.  $x$  and  $y$  are returned as integers. The generated private key  $x$  is in the range  $[1, q-1]$ , and the public key is in the range  $[1, p-1]$ .

**Process:**

1.  $N = \text{len}(q); L = \text{len}(p)$ .  

Comment: Check that the  $(L, N)$  pair is specified in Section 4.2.
2. If the  $(L, N)$  pair is invalid, then return an **ERROR** indicator, *Invalid\_x*, and *Invalid\_y*.
3. *requested\_security\_strength* = the security strength associated with the  $(L, N)$  pair; see SP 800-57.
4. Obtain a string of  $N+64$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_x*, and *Invalid\_y*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6.  $x = (c \bmod (q-1)) + 1$ .  

Comment:  $0 \leq c \bmod (q-1) \leq q-2$  and implies that  $1 \leq x \leq q-1$ .
7.  $y = g^x \bmod p$ .
8. Return **SUCCESS**,  $x$ , and  $y$ .

### B.1.2 Key Pair Generation by Testing Candidates

In this method, a random number is obtained and tested to determine that it will produce a value of  $x$  in the correct range. If  $x$  is out-of-range, another random number is obtained (i.e., the process is iterated until an acceptable value of  $x$  is obtained).

The following process or its equivalent may be used to generate an FFC key pair.

**Input:**

- $(p, q, g)$  The subset of the domain parameters that are used for this process.  $p$ ,  $q$  and  $g$  **shall** either be provided as integers during input, or **shall** be converted to integers prior to use.

**Output:**

1. *status*      The status returned from the key pair generation process. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(x, y)$       The generated private and public keys. If an error is encountered during the generation process, invalid values for  $x$  and  $y$  **should** be returned, as represented by *Invalid\_x* and *Invalid\_y* in the following specification.  $x$  and  $y$  are returned as integers. The generated private key  $x$  is in the range  $[1, q-1]$ , and the public key is in the range  $[1, p-1]$ .

**Process:**

1.  $N = \text{len}(q)$ ;  $L = \text{len}(p)$ .  

Comment: Check that the  $(L, N)$  pair is specified in Section 4.2.
2. If the  $(L, N)$  pair is invalid, then return an **ERROR** indication, *Invalid\_x*, and *Invalid\_y*.
3. *requested\_security\_strength* = the security strength associated with the  $(L, N)$  pair; see SP 800-57.
4. Obtain a string of  $N$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_x*, and *Invalid\_y*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6. If  $(c > q-2)$ , then go to step 4.
7.  $x = c + 1$ .
8.  $y = g^x \bmod p$ .
9. Return **SUCCESS**,  $x$ , and  $y$ .

## B.2 FFC Per-Message Secret Number Generation

DSA requires the generation of a new random number  $k$  for each message to be signed. Two methods are provided for the generation of  $k$ ; one of these two methods or another approved method **shall** be used.

The valid values of  $N$  are provided in Section 4.2. Let **inverse**( $k, q$ ) be a function that computes the inverse of a (non-negative) integer  $k$  with respect to multiplication modulo the prime number  $q$ . A technique for computing the inverse is provided in Appendix C.1.

### B.2.1 Per-Message Secret Number Generation Using Extra Random Bits

In this method, 64 more bits are requested from the RBG than are needed for  $k$  so that bias produced by the mod function in step 6 is not readily apparent.

The following process or its equivalent may be used to generate a per-message secret number.

**Input:**

$(p, q, g)$  DSA domain parameters that are generated as specified in Section 4.3.1.

**Output:**

1.  $status$  The status returned from the secret number generation process. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(k, k^{-1})$  The per-message secret number  $k$  and its mod  $q$  inverse,  $k^{-1}$ . If an error is encountered during the generation process, invalid values for  $k$  and  $k^{-1}$  **should** be returned, as represented by *Invalid\_k* and *Invalid\_k\_inverse* in the following specification.  $k$  and  $k^{-1}$  are in the range  $[1, q-1]$ .

**Process:**

1.  $N = \text{len}(q)$ ;  $L = \text{len}(p)$ .  

Comment: Check that the  $(L, N)$  pair is specified in Section 4.2.
2. If the  $(L, N)$  pair is invalid, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
3.  $requested\_security\_strength$  = the security strength associated with the  $(L, N)$  pair; see SP 800-57.
4. Obtain a string of  $N+64$  *returned\_bits* from an **RBG** with a security strength of  $requested\_security\_strength$  or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6.  $k = (c \bmod (q-1)) + 1$ .
7.  $(status, k^{-1}) = \text{inverse}(k, q)$ .
8. Return  $status$ ,  $k$ , and  $k^{-1}$ .

### B.2.2 Per-Message Secret Number Generation by Testing Candidates

In this method, a random number is obtained and tested to determine that it will produce a value of  $k$  in the correct range. If  $k$  is out-of-range, another random number is obtained (i.e., the process is iterated until an acceptable value of  $k$  is obtained).

The following process or its equivalent may be used to generate a per-message secret number.

**Input:**

$(p, q, g)$  DSA domain parameters that are generated as specified in Section 4.3.1.

**Output:**

1. *status* The status returned from the secret number generation process. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(k, k^{-1})$  The per-message secret number  $k$  and its inverse,  $k^{-1}$ . If an error is encountered during the generation process, invalid values for  $k$  and  $k^{-1}$  **should** be returned, as represented by *Invalid\_k* and *Invalid\_k\_inverse* in the following specification.  $k$  and  $k^{-1}$  are in the range  $[1, q-1]$ .

**Process:**

1.  $N = \text{len}(q)$ ;  $L = \text{len}(p)$ .  

Comment: Check that the  $(L, N)$  pair is specified in Section 4.2).
2. If the  $(L, N)$  pair is invalid, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
3. *requested\_security\_strength* = the security strength associated with the  $(L, N)$  pair; see SP 800-57.
4. Obtain a string of  $N$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6. If  $(c > q-2)$ , then go to step 4.
7.  $k = c + 1$ .
8.  $(\text{status}, k^{-1}) = \text{inverse}(k, q)$ .
9. Return *status*,  $k$ , and  $k^{-1}$ .

### B.3 IFC Key Pair Generation

#### B.3.1 Criteria for IFC Key Pairs

Key pairs for IFC consist of a public key  $(n, e)$ , and a private key  $(n, d)$ , where  $n$  is the modulus and is the product of two prime numbers  $p$  and  $q$ . The security of IFC depends on the quality and secrecy of these primes and the private exponent  $d$ . The primes  $p$  and  $q$  **shall** be generated using

one of the following methods:

A. Both  $p$  and  $q$  are randomly generated prime numbers (Random Primes), where  $p$  and  $q$  **shall** both be either :

1. Provable primes (see Appendix B.3.2), or
2. Probable primes (see Appendix B.3.3).

Using methods 1 and 2,  $p$  and  $q$  with lengths of 1024 or 1536 bits may be generated;  $p$  and  $q$  with lengths of 512 bits **shall not** be generated using these methods. Instead,  $p$  and  $q$  with lengths of 512 bits **shall** be generated using the conditions based on auxiliary primes (see Appendices B.3.4, B.3.5, or B.3.6).

B. Both  $p$  and  $q$  are randomly generated prime numbers that satisfy the following additional conditions (Primes with Conditions):

- $(p-1)$  has a prime factor  $p_1$
- $(p+1)$  has a prime factor  $p_2$
- $(q-1)$  has a prime factor  $q_1$
- $(q+1)$  has a prime factor  $q_2$

where  $p_1, p_2, q_1$  and  $q_2$  are called auxiliary primes of  $p$  and  $q$ .

Using this method, one of the following cases **shall** apply:

1. The primes  $p_1, p_2, q_1, q_2, p$  and  $q$  **shall** all be provable primes (see Appendix B.3.4),
2. The primes  $p_1, p_2, q_1$  and  $q_2$  **shall** be provable primes, and the primes  $p$  and  $q$  **shall** be probable primes (see Appendix B.3.5), or
3. The primes  $p_1, p_2, q_1, q_2, p$  and  $q$  **shall** all be probable primes (see Appendix B.3.6).

The minimum lengths for each of the auxiliary primes  $p_1, p_2, q_1$  and  $q_2$  are dependent on  $nlen$ , where  $nlen$  is the length of the modulus  $n$  in bits. Note that  $nlen$  is also called the key size. The lengths of the auxiliary primes may be fixed or randomly chosen, subject to the restrictions in Table B.1. The maximum length is determined by  $nlen$  (the sum of the length of each auxiliary prime pair) and whether the primes  $p$  and  $q$  are probable primes or provable primes (e.g., for the auxiliary prime pair  $p_1$  and  $p_2$ ,  $\text{len}(p_1) + \text{len}(p_2)$  **shall** be less than a value determined by  $nlen$ , whether  $p_1$  and  $p_2$  are generated to be probable or provable primes)<sup>3</sup>.

---

<sup>3</sup> For the probable primes  $p$  and  $q$ :  $\text{len}(p_1) + \text{len}(p_2) < \text{len}(p) - \log_2(\text{len}(p)) - 6$ ; similarly for  $\text{len}(q_1) + \text{len}(q_2)$  and  $\text{len}(q)$ . For the provable primes  $p$  and  $q$ :  $\text{len}(p_1) + \text{len}(p_2) < \text{len}(p)/2 - \log_2(\text{len}(p)) - 7$ ; similarly for  $\text{len}(q_1) + \text{len}(q_2)$

**Table B.1. Minimum and maximum lengths of  $p_1, p_2, q_1$  and  $q_2$**

<i>nlen</i>	Min. length of auxiliary primes $p_1, p_2, q_1$ and $q_2$	Max. length of $\text{len}(p_1) + \text{len}(p_2)$ and $\text{len}(q_1) + \text{len}(q_2)$	
		<i>p, q</i> Probable primes	<i>p, q</i> Provable primes
1024	> 100 bits	< 496 bits	< 239 bits
2048	> 140 bits	< 1007 bits	< 494 bits
3072	> 170 bits	< 1518 bits	< 750 bits

For different values of *nlen* (i.e., different key sizes), the methods allowed for the generation of *p* and *q* are specified in Table B.2.

**Table B.2. Allowable Prime Generation Methods**

<i>nlen</i>	Random Primes	Primes with Conditions
1024	No	Yes
2048	Yes	Yes
3072	Yes	Yes

In addition, all IFC keys **shall** meet the following criteria in order to conform to FIPS 186-4:

1. The public exponent *e* **shall** be selected with the following constraints:
  - (a) The public verification exponent *e* **shall** be selected prior to generating the primes *p* and *q*, and the private signature exponent *d*.
  - (b) The exponent *e* **shall** be an odd positive integer such that:

$$2^{16} < e < 2^{256}.$$

Note that the value of *e* may be any value that meets constraint 1(b), i.e., *e* may be either a fixed value or a random value.

2. The primes *p* and *q* **shall** be selected with the following constraints:
  - (a)  $(p-1)$  and  $(q-1)$  **shall** be relatively prime to the public exponent *e*.
  - (b) The private prime factor *p* **shall** be selected randomly and **shall** satisfy

---

and  $\text{len}(q)$ . In each case,  $\text{len}(p) = \text{len}(q) = nlen/2$ .

$(\sqrt{2})(2^{(nlen/2)-1}) \leq p \leq (2^{nlen/2} - 1)$ , where  $nlen$  is the appropriate length for the desired *security\_strength*.

(c) The private prime factor  $q$  **shall** be selected randomly and **shall** satisfy  $(\sqrt{2})(2^{(nlen/2)-1}) \leq q \leq (2^{nlen/2} - 1)$ , where  $nlen$  is the appropriate length for the desired *security\_strength*.

(d)  $|p - q| > 2^{(nlen/2) - 100}$ .

3. The private signature exponent  $d$  **shall** be selected with the following constraints after the generation of  $p$  and  $q$ :

(a) The exponent  $d$  **shall** be a positive integer value such that  $2^{nlen/2} < d < \text{LCM}(p-1, q-1)$ , and

(b)  $d = e^{-1} \text{ mod } (\text{LCM}(p-1, q-1))$ .

That is, the inequality in (a) holds, and  $1 \equiv (ed) \text{ (mod } \text{LCM}(p-1, q-1))$ .

In the extremely rare event that  $d \leq 2^{nlen/2}$ , then new values for  $p$ ,  $q$  and  $d$  **shall** be determined. A different value of  $e$  may be used, although this is not required.

Any hash function used during the generation of the key pair **shall** be approved (i.e., specified in FIPS 180).

### B.3.2 Generation of Random Primes that are Provably Prime

An approved method that satisfies the constraints of Appendix B.3.1 **shall** be used for the generation of IFC random primes  $p$  and  $q$  that are provably prime (see case A.1). One such method is provided in Appendix B.3.2.1 and B.3.2.2. For this method, a random seed is initially required (see Appendix B.3.2.1); the length of the seed is equal to twice the security strength associated with the modulus  $n$ . After the seed is obtained, the primes can be generated (see Appendix B.3.2.2).

#### B.3.2.1 Get the Seed

The following process or its equivalent **shall** be used to generate the seed for this method.

##### Input:

$nlen$  The intended bit length of the modulus  $n$ .

##### Output:

$status$  The status to be returned, where  $status$  is either **SUCCESS** or **FAILURE**.

$seed$  The seed. If  $status = \text{FAILURE}$ , a value of zero is returned as the  $seed$ .

##### Process:

1. If  $nlen$  is not valid (see Section 5.1), then Return (**FAILURE**, 0).

2. Let *security\_strength* be the security strength associated with *nlen*, as specified in SP 800-57, Part 1.
3. Obtain a string *seed* of  $(2 * \text{security\_strength})$  bits from an **RBG** that supports the *security\_strength*.
4. Return (**SUCCESS**, *seed*).

### B.3.2.2 Construction of the Provable Primes *p* and *q*

The following process or its equivalent **shall** be used to construct the random primes *p* and *q* (to be used as factors of the RSA modulus *n*) that are provably prime:

#### Input:

- |             |   |
|-------------|---|
| <i>nlen</i> | The intended bit length of the modulus <i>n</i> .       |
| <i>e</i>    | The public verification exponent.                       |
| <i>seed</i> | The seed obtained using the method in Appendix B.3.2.1. |

#### Output:

- |                       |   |
|-----------------------|---|
| <i>status</i>         | The status of the generation process, where <i>status</i> is either <b>SUCCESS</b> or <b>FAILURE</b> . When <b>FAILURE</b> is returned, zero values <b>shall</b> be returned as the other parameters. |
| <i>p</i> and <i>q</i> | The private prime factors of <i>n</i> .   |

#### Process:

1. If *nlen* is neither 2048 nor 3072, then return (**FAILURE**, 0, 0).
2. If  $((e \leq 2^{16}) \text{ OR } (e \geq 2^{256}) \text{ OR } (e \text{ is not odd}))$ , then return (**FAILURE**, 0, 0).
3. Set the value of *security\_strength* in accordance with the value of *nlen*, as specified in SP 800-57, Part 1.
4. If  $(\text{len}(\text{seed}) \neq 2 * \text{security\_strength})$ , then return (**FAILURE**, 0, 0).
5. *working\_seed* = *seed*.
6. Generate *p*:
  - 6.1 Using  $L = nlen/2$ ,  $N_1 = 1$ ,  $N_2 = 1$ , *first\_seed* = *working\_seed* and *e*, use the provable prime construction method in Appendix C.10 to obtain *p* and *pseed*. If **FAILURE** is returned, then return (**FAILURE**, 0, 0).
  - 6.2 *working\_seed* = *pseed*.



7. Generate  $q$ :
  - 7.1 Using  $L = nlen/2$ ,  $N_1 = 1$ ,  $N_2 = 1$ ,  $first\_seed = working\_seed$  and  $e$ , use the provable prime construction method in Appendix C.10 to obtain  $q$  and  $qseed$ . If **FAILURE** is returned, then return (**FAILURE**, 0, 0).
  - 7.2  $working\_seed = qseed$ .
8. If  $(|p - q| \leq 2^{nlen/2 - 100})$ , then go to step 7.
9. Zeroize the internally generated seeds:
  - 9.1  $pseed = 0$ ;
  - 9.2  $qseed = 0$ ;
  - 9.3  $working\_seed = 0$ .
10. Return (**SUCCESS**,  $p$ ,  $q$ ).

### B.3.3 Generation of Random Primes that are Probably Prime

An approved method that satisfies the constraints of Appendix B.3.1 **shall** be used for the generation of IFC random primes  $p$  and  $q$  that are probably prime (see case A.2).

The following process or its equivalent **shall** be used to construct the random probable primes  $p$  and  $q$  (to be used as factors of the RSA modulus  $n$ ):

#### Input:

- |        |  |
|--------|--|
| $nlen$ | The intended bit length of the modulus $n$ . |
| $e$    | The public verification exponent.            |

#### Output:

- |             |   |
|-------------|---|
| $status$    | The status of the generation process, where $status$ is either <b>SUCCESS</b> or <b>FAILURE</b> .                         |
| $p$ and $q$ | The private prime factors of $n$ . When <b>FAILURE</b> is returned, zero values <b>shall</b> be returned as $p$ and $q$ . |

#### Process:

1. If  $nlen$  is neither 2048 nor 3072, return (**FAILURE**, 0, 0).
2. If  $((e \leq 2^{16}) \text{ OR } (e \geq 2^{256}) \text{ OR } (e \text{ is not odd}))$ , then return (**FAILURE**, 0, 0).
3. Set the value of  $security\_strength$  in accordance with the value of  $nlen$ , as specified in SP 800-57, Part 1.
4. Generate  $p$ :
  - 4.1  $i = 0$ .

- 4.2 Obtain a string  $p$  of  $(nlen/2)$  bits from an **RBG** that supports the *security\_strength*.
- 4.3 If ( $p$  is not odd), then  $p = p + 1$ .
- 4.4 If  $((p < (\sqrt{2})(2^{(nlen/2)-1}))$ ), then go to step 4.2.
- 4.5 If (**GCD**( $p-1, e$ ) = 1), then
  - 4.5.1 Test  $p$  for primality as specified in Appendix C.3, using an appropriate value from Table C-2 or C-3 in Appendix C.3 as the number of iterations.
  - 4.5.2 If  $p$  is **PROBABLY PRIME**, then go to step 5.
- 4.6  $i = i + 1$ .
- 4.7 If  $(i \geq 5(nlen/2))$ , then return (**FAILURE**, 0, 0)  
Else go to step 4.2.
5. Generate  $q$ :
  - 5.1  $i = 0$ .
  - 5.2 Obtain a string  $q$  of  $(nlen/2)$  bits from an **RBG** that supports the *security\_strength*
  - 5.3 If ( $q$  is not odd), then  $q = q + 1$ .
  - 5.4 If  $(|p - q| \leq 2^{nlen/2 - 100})$ , then go to step 5.2.
  - 5.5 If  $((q < (\sqrt{2})(2^{(nlen/2)-1}))$ ), then go to step 5.2.
  - 5.6 If (**GCD**( $q-1, e$ ) = 1) then
    - 5.6.1 Test  $q$  for primality as specified in Appendix C.3, using an appropriate value from Table C-2 or C-3 in Appendix C.3 as the number of iterations.
    - 5.6.2 If  $q$  is **PROBABLY PRIME**, then return (**SUCCESS**,  $p, q$ ).
  - 5.7  $i = i + 1$ .
  - 5.8 If  $(i \geq 5(nlen/2))$ , then return (**FAILURE**, 0, 0)  
Else go to step 5.2.

#### **B.3.4 Generation of Provable Primes with Conditions Based on Auxiliary Provable Primes**

This section specifies an approved method for the generation of the IFC primes  $p$  and  $q$  with the additional conditions specified in Appendix B.3.1, case B.1, where  $p, p_1, p_2, q, q_1$  and  $q_2$  are all provable primes. For this method, a random seed is initially required (see Appendix B.3.2.1); the length of the seed is equal to twice the security strength associated with the modulus  $n$ . After the first seed is obtained, the primes can be generated.

Let  $bitlen_1$ ,  $bitlen_2$ ,  $bitlen_3$ , and  $bitlen_4$  be the bit lengths for  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$ , respectively, in accordance with Table B.1. The following process or its equivalent **shall** be used to generate the provable primes:

**Input:**

- $nlen$       The intended bit length of the modulus  $n$ .
- $e$           The public verification exponent.
- $seed$       The seed obtained using the method in Appendix B.3.2.1.

**Output:**

- $status$       The status of the generation process, where  $status$  is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned then zeros **shall** be returned as the values for  $p$  and  $q$ .
- $p$  and  $q$       The private prime factors of  $n$ .

**Process:**

1. If  $nlen$  is neither 1024, 2048, nor 3072, then return (**FAILURE**, 0, 0).
2. If  $((e \leq 2^{16}) \text{ OR } (e \geq 2^{256}) \text{ OR } (e \text{ is not odd}))$ , then return (**FAILURE**, 0, 0).
3. Set the value of  $security\_strength$  in accordance with the value of  $nlen$ , as specified in SP 800-57, Part 1.
4. If  $(len(seed) \neq 2 * security\_strength)$ , then return (**FAILURE**, 0, 0).
5.  $working\_seed = seed$ .
6. Generate  $p$ :
  - 6.1 Using  $L = nlen/2$ ,  $N_1 = bitlen_1$ ,  $N_2 = bitlen_2$ ,  $firstseed = working\_seed$  and  $e$ , use the provable prime construction method in Appendix C.10 to obtain  $p$ ,  $p_1$ ,  $p_2$  and  $pseed$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
  - 6.2  $working\_seed = pseed$ .
7. Generate  $q$ :
  - 7.1 Using  $L = nlen/2$ ,  $N_1 = bitlen_3$ ,  $N_2 = bitlen_4$  and  $firstseed = working\_seed$  and  $e$ , use the provable prime construction method in Appendix C.10 to obtain  $q$ ,  $q_1$ ,  $q_2$  and  $qseed$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
  - 7.2  $working\_seed = qseed$ .
8. If  $(|p - q| \leq 2^{nlen/2 - 100})$ , then go to step 7.
9. Zeroize the internally generated seeds:
  - 9.1  $pseed = 0$ .

9.2  $q_{seed} = 0$ .

9.3  $working\_seed = 0$ .

10. Return (**SUCCESS**,  $p$ ,  $q$ ).

### B.3.5 Generation of Probable Primes with Conditions Based on Auxiliary Provable Primes

This section specifies an approved method for the generation of the IFC primes  $p$  and  $q$  with the additional conditions specified in Appendix B.3.1, case B.2, where  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  are provably prime, and  $p$  and  $q$  are probably prime. For this method, a random seed is initially required (see Appendix B.3.2.1); the length of the seed is equal to twice the security strength associated with the modulus  $n$ . After the first seed is obtained, the primes can be generated.

Let  $bitlen_1$ ,  $bitlen_2$ ,  $bitlen_3$ , and  $bitlen_4$  be the bit lengths for  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$ , respectively in accordance with Table B.1. The following process or its equivalent **shall** be used to construct  $p$  and  $q$ .

#### Input:

$nlen$       The intended bit length of the modulus  $n$ .  
 $e$           The public verification exponent.  
 $seed$       The seed obtained using the method in Appendix B.3.2.1.

#### Output:

$status$       The status of the generation process, where  $status$  is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned then zeros **shall** be returned as the values for  $p$  and  $q$ .  
 $p$  and  $q$       The private prime factors of  $n$ .

#### Process:

1. If  $nlen$  is neither 1024, 2048, nor 3072, then return (**FAILURE**, 0, 0).
2. If  $((e \leq 2^{16}) \text{ OR } (e \geq 2^{256}) \text{ OR } (e \text{ is not odd}))$ , then return (**FAILURE**, 0, 0).
3. Set the value of  $security\_strength$  in accordance with the value of  $nlen$ , as specified in SP 800-57, Part 1.
4. If  $(len(seed) \neq 2 * security\_strength)$ , then return (**FAILURE**, 0, 0).

Comment: Generate four primes  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  that are provably prime.

5. Generate  $p$ :
  - 5.1 Using  $bitlen_1$  as the length, and  $seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $p_1$  and  $prime\_seed$ . If **FAILURE** is returned, the return (**FAILURE**, 0, 0).
  - 5.2 Using  $bitlen_2$  as the length, and  $prime\_seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $p_2$  and a new value for  $prime\_seed$ . If **FAILURE** is returned, the return (**FAILURE**, 0, 0).
  - 5.3 Generate a prime  $p$  using the routine in Appendix C.9 with inputs of  $p_1$ ,  $p_2$ ,  $nlen$ ,  $e$  and  $security\_strength$ , also obtaining  $X_p$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
6. Generate  $q$ :
  - 6.1 Using  $bitlen_3$  as the length, and  $prime\_seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $q_1$  and a new value for  $prime\_seed$ . If **FAILURE** is returned, the return (**FAILURE**, 0, 0).
  - 6.2 Using  $bitlen_4$  as the length, and  $prime\_seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $q_2$  and a new value for  $prime\_seed$ . If **FAILURE** is returned, the return (**FAILURE**, 0, 0).
  - 6.3 Generate a prime  $q$  using the routine in Appendix C.9 with inputs of  $q_1$ ,  $q_2$ ,  $nlen$ ,  $e$  and  $security\_strength$ , also obtaining  $X_q$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
7. If  $((|p - q| \leq 2^{nlen/2 - 100}) \text{ OR } (|X_p - X_q| \leq 2^{nlen/2 - 100}))$ , then go to step 6.
8. Zeroize the internally generated that are not returned:
  - 8.1  $X_p = 0$ .
  - 8.2  $X_q = 0$ .
  - 8.3  $prime\_seed = 0$ .
  - 8.4  $p_1 = 0$ .
  - 8.5  $p_2 = 0$ .
  - 8.6  $q_1 = 0$ .
  - 8.7  $q_2 = 0$ .
9. Return (**SUCCESS**,  $p$ ,  $q$ ).

### B.3.6 Generation of Probable Primes with Conditions Based on Auxiliary Probable Primes

An approved method that satisfies the constraints of Appendix B.3.1 **shall** be used for the generation of IFC primes  $p$  and  $q$  that are probably prime and meet the additional constraints of Appendix B.3.1 (see case B.3). For this case, the prime factors  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  are also probably prime.

Four random numbers  $X_{p1}$ ,  $X_{p2}$ ,  $X_{q1}$  and  $X_{q2}$  are generated, from which the prime factors  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  are determined.  $p_1$  and  $p_2$ , and an additional random number  $X_p$  are then used to determine  $p$ , and  $q_1$  and  $q_2$  and a random number  $X_q$  are used to obtain  $q$ . Let  $bitlen_1$ ,  $bitlen_2$ ,  $bitlen_3$ , and  $bitlen_4$  be the bit lengths for  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$ , respectively chosen in accordance with Table B.1.

The following process or its equivalent **shall** be used to generate  $p$  and  $q$ :

#### Input:

$nlen$                       The intended bit length of the modulus  $n$ .  
 $e$                               The public verification exponent.

#### Output:

$status$                       The status of the generation process, where  $status$  is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned then zeros **shall** be returned as the values for  $p$  and  $q$ .  
 $p$  and  $q$                       The private prime factors of  $n$ .

#### Process:

1. If  $nlen$  is neither 1024, 2048, nor 3072, then return (**FAILURE**, 0, 0).
2. If  $((e \leq 2^{16}) \text{ OR } (e \geq 2^{256}) \text{ OR } (e \text{ is not odd}))$ , then return (**FAILURE**, 0, 0).
3. Set the value of  $security\_strength$  in accordance with the value of  $nlen$ , as specified in SP 800-57, Part 1.
4. Generate  $p$ :
  - 4.1 Generate an odd integer  $X_{p1}$  of length  $bitlen_1$  bits, and a second odd integer  $X_{p2}$  of length  $bitlen_2$  bits, using an approved random number generator that supports the  $security\_strength$ .
  - 4.2 Sequentially search successive odd integers, starting at  $X_{p1}$  until the first probable prime  $p_1$  is found. Candidate integers **shall** be tested for primality as specified in Appendix C.3. Repeat the process to find  $p_2$ , starting at  $X_{p2}$ . The probable primes  $p_1$  and  $p_2$  **shall** be the first integers that pass the primality test.

- 4.3 Generate a prime  $p$  using the routine in Appendix C.9 with inputs of  $p_1, p_2, nlen, e$  and *security\_strength*, also obtaining  $X_p$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
5. Generate  $q$ :
  - 5.1 Generate an odd integer  $X_{q1}$  of length  $bitlen_3$  bits, and a second odd integer  $X_{q2}$  of length  $bitlen_4$  bits, using an approved random number generator that supports the *security\_strength*.
  - 5.2 Sequentially search successive odd integers, starting at  $X_{q1}$  until the first probable prime  $q_1$  is found. Candidate integers **shall** be tested for primality as specified in Appendix C.3. Repeat the process to find  $q_2$ , starting at  $X_{q2}$ . The probable primes  $q_1$  and  $q_2$  **shall** be the first integers that pass the primality test.
  - 5.3 Generate a prime  $q$  using the routine in Appendix C.9 with inputs of  $q_1, q_2, nlen, e$  and *security\_strength*, also obtaining  $X_q$ . If **FAILURE** is returned, return (**FAILURE**, 0, 0).
6. If  $((|X_p - X_q| \leq 2^{nlen/2 - 100}) \text{ OR } (|p - q| \leq 2^{nlen/2 - 100}))$ , then go to step 5.
7. Zeroize the internally generated values that are not returned:
  - 7.1  $X_p = 0$ .
  - 7.2  $X_q = 0$ .
  - 7.3  $X_{p1} = 0$ .
  - 7.4  $X_{p2} = 0$ .
  - 7.5  $X_{q1} = 0$ .
  - 7.6  $X_{q2} = 0$ .
  - 7.7  $p_1 = 0$ .
  - 7.8  $p_2 = 0$ .
  - 7.9  $q_1 = 0$ .
  - 7.10  $q_2 = 0$ .
8. Return (**SUCCESS**,  $p, q$ ).

#### B.4 ECC Key Pair Generation

An ECC key pair  $d$  and  $Q$  is generated for a set of domain parameters  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$ . Two methods are provided for the generation of the ECC private key  $d$  and public key  $Q$ ; one of these two methods **shall** be used to generate  $d$  and  $Q$ .

Prior to generating ECDSA key pairs, assurance of the validity of the domain parameters ( $q$ ,  $FR$ ,  $a$ ,  $b$ ,  $\{, domain\_parameter\_seed\}$ ,  $G$ ,  $n$ ,  $h$ ) **shall** have been obtained as specified in Section 3.1.

For ECDSA, the valid bit-lengths of  $n$  are provided in Section 6.1.1. See ANS X9.62 for definitions of the elliptic curve math and the conversion routines.

#### B.4.1 Key Pair Generation Using Extra Random Bits

In this method, 64 more bits are requested from the RBG than are needed for  $d$  so that bias produced by the mod function in step 6 is negligible.

The following process or its equivalent may be used to generate an ECC key pair.

##### Input:

1.  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$   
The domain parameters that are used for this process.  $n$  is a prime number, and  $G$  is a point on the elliptic curve.

##### Output:

1.  $status$  The status returned from the key pair generation procedure. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(d, Q)$  The generated private and public keys. If an error is encountered during the generation process, invalid values for  $d$  and  $Q$  **should** be returned, as represented by *Invalid\_d* and *Invalid\_Q* in the following specification.  $d$  is an integer, and  $Q$  is an elliptic curve point. The generated private key  $d$  is in the range  $[1, n-1]$ .

##### Process:

1.  $N = \text{len}(n)$ .  

Comment: Check that  $N$  is included in Table 1 of Section 6.1.1.
2. If  $N$  is invalid, then return an **ERROR** indication, *Invalid\_d*, and *Invalid\_Q*.
3.  $requested\_security\_strength$  = the security strength associated with  $N$ ; see SP 800-57, Part 1.
4. Obtain a string of  $N+64$  *returned\_bits* from an **RBG** with a security strength of  $requested\_security\_strength$  or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_d*, and *Invalid\_Q*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6.  $d = (c \bmod (n-1)) + 1$ .



7.  $Q = dG$ .
8. Return **SUCCESS**,  $d$ , and  $Q$ .

#### B.4.2 Key Pair Generation by Testing Candidates

In this method, a random number is obtained and tested to determine that it will produce a value of  $d$  in the correct range. If  $d$  is out-of-range, another random number is obtained (i.e., the process is iterated until an acceptable value of  $d$  is obtained).

The following process or its equivalent may be used to generate an ECC key pair.

##### Input:

1.  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$   
The domain parameters that are used for this process.  $n$  is a prime number, and  $G$  is a point on the elliptic curve.

##### Output:

1. *status* The status returned from the key pair generation procedure. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(d, Q)$  The generated private and public keys. If an error is encountered during the generation process, invalid values for  $d$  and  $Q$  **should** be returned, as represented by *Invalid\_d* and *Invalid\_Q* in the following specification.  $d$  is an integer, and  $Q$  is an elliptic curve point. The generated private key  $d$  is in the range  $[1, n-1]$ .

##### Process:

1.  $N = \text{len}(n)$ .  

Comment: Check that  $N$  is included in Table 1 of Section 6.1.1.
2. If  $N$  is invalid, then return an **ERROR** indication, *Invalid\_d*, and *Invalid\_Q*.
3. *requested\_security\_strength* = the security strength associated with  $N$ ; see SP 800-57, Part 1.
4. Obtain a string of  $N$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_d*, and *Invalid\_Q*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6. If  $(c > n-2)$ , then go to step 4.
7.  $d = c + 1$ .

8.  $Q = dG$ .
9. Return **SUCCESS**,  $d$ , and  $Q$ .

## B.5 ECC Per-Message Secret Number Generation

ECDSA requires the generation of a new random number  $k$  for each message to be signed. Two methods are provided for the generation of  $k$ ; one of these two methods or another approved method **shall** be used.

The valid values of  $n$  are provided in Section 6.1.1. See ANS X9.62 for definitions of the elliptic curve math and the conversion routines.

Let **inverse**( $k, n$ ) be a function that computes the inverse of a (non-negative) integer  $k$  with respect to multiplication modulo the prime number  $n$ . A technique for computing the inverse is provided in Appendix C.1.

### B.5.1 Per-Message Secret Number Generation Using Extra Random Bits

In this method, 64 more bits are requested from the RBG than are needed for  $k$  so that bias produced by the mod function in step 6 is not readily apparent.

The following process or its equivalent may be used to generate a per-message secret number.

#### Input:

1.  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$   
The domain parameters that are used for this process.  $n$  is a prime number, and  $G$  is a point on the elliptic curve.

#### Output:

1.  $status$  The status returned from the key pair generation procedure. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(k, k^{-1})$  The generated secret number  $k$  and its inverse  $k^{-1}$ . If an error is encountered during the generation process, invalid values for  $k$  and  $k^{-1}$  **should** be returned, as represented by *Invalid\_k* and *Invalid\_k\_inverse* in the following specification.  $k$  and  $k^{-1}$  are integers in the range  $[1, n-1]$ .

#### Process:

1.  $N = \text{len}(n)$ .  
  
Comment: Check that  $N$  is included in Table 1 of Section 6.1.1.
2. If  $N$  is invalid, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.

3. *requested\_security\_strength* = the security strength associated with *N*; see SP 800-57, Part 1.
4. Obtain a string of  $N+64$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
5. Convert *returned\_bits* to the non-negative integer *c* (see Appendix C.2.1).
6.  $k = (c \bmod (n-1)) + 1$ .
7.  $(status, k^{-1}) = \text{inverse}(k, n)$ .
8. Return *status*, *k*, and  $k^{-1}$ .

### B.5.2 Per-Message Secret Number Generation by Testing Candidates

In this method, a random number is obtained and tested to determine that it will produce a value of *k* in the correct range. If *k* is out-of-range, another random number is obtained (i.e., the process is iterated until an acceptable value of *k* is obtained).

The following process or its equivalent may be used to generate a per-message secret number.

#### Input:

1.  $(q, FR, a, b \{, domain\_parameter\_seed\}, G, n, h)$

The domain parameters that are used for this process. *n* is a prime number, and *G* is a point on the elliptic curve.

#### Output:

1. *status*      The status returned from the key pair generation procedure. The status will indicate **SUCCESS** or an **ERROR**.
2.  $(k, k^{-1})$       The generated secret number *k* and its inverse  $k^{-1}$ . If an error is encountered during the generation process, invalid values for *k* and  $k^{-1}$  **should** be returned, as represented by *Invalid\_k* and *Invalid\_k\_inverse* in the following specification. *k* and  $k^{-1}$  are integers in the range  $[1, n-1]$ .

#### Process:

1.  $N = \text{len}(n)$ .

Comment: Check that *N* is included in Table 1 of Section 6.1.1.

2. If *N* is not included in Table 1, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.

3. *requested\_security\_strength* = the security strength associated with  $N$ ; see SP 800-57, Part 1.
4. Obtain a string of  $N$  *returned\_bits* from an **RBG** with a security strength of *requested\_security\_strength* or more. If an **ERROR** indication is returned, then return an **ERROR** indication, *Invalid\_k*, and *Invalid\_k\_inverse*.
5. Convert *returned\_bits* to the (non-negative) integer  $c$  (see Appendix C.2.1).
6. If  $(c > n-2)$ , then go to step 4.
7.  $k = c + 1$ .
8.  $(status, k^{-1}) = \mathbf{inverse}(k, n)$ .
9. Return *status*,  $k$ , and  $k^{-1}$ .

## Appendix C: Generation of Other Quantities

This appendix contains routines for supplementary processes required for the implementation of this Standard. Appendix C.1 is needed to produce the inverse of the per-message secret  $k$  (see Section 4.5, and Appendices B.2.1, B.2.2, B.5.1 and B.5.2) and the inverse of the signature portion  $s$  that is used during signature verification (see Section 4.7). The routines in Appendix C.2 are required to convert between bit strings and integers where required in implementing this Standard. Appendix C.3 contains probabilistic primality tests to be used during the generation of DSA domain parameters and RSA key pairs. Appendices C.4 and C.5 contain algorithms required during the Lucas probabilistic primality test of Appendix C.3.3 to check for a perfect square and to compute the Jacobi symbol. Appendix C.6 contains the Shawe-Taylor algorithm for the construction of primes. Appendix C.7 provides a process to perform trial division, as required by the random prime generation routine in Appendix C.6. The sieve procedure in Appendix C.8 is needed by the trial division routine in Appendix C.7. The trial division process in Appendix C.7 and the sieve procedure in Appendix C.8 have been extracted from ANS X9.80, *Prime Number Generation, Primality Testing, and Primality Certificates*. Appendix C.9 is required during the generation of RSA key pairs. Appendix C.10 provides a method for constructing provable primes for RSA (see Appendix B.3.2.2 and B.3.4).

### C.1 Computation of the Inverse Value

This algorithm or an algorithm that produces an equivalent result **shall** be used to compute the multiplicative inverse  $z^{-1} \bmod a$ , where  $0 < z < a$ ,  $0 < z^{-1} < a$ , and  $a$  is a prime number. In this Standard,  $z$  is either  $k$  or  $s$ , and  $a$  is either  $q$  or  $n$ .

#### Input:

1.  $z$  The value to be inverted mod  $a$  (i.e., either  $k$  or  $s$ ).
2.  $a$  The domain parameter and (prime) modulus (i.e., either  $q$  or  $n$ ).

#### Output:

1.  $status$  The status returned from this function, where the  $status$  is either **SUCCESS** or **ERROR**.
2.  $z^{-1}$  The multiplicative inverse of  $z \bmod a$ , if it exists.

#### Process:

1. Verify that  $a$  and  $z$  are positive integers such that  $z < a$ ; if not, return an **ERROR** indication.
2. Set  $i = a$ ,  $j = z$ ,  $y_2 = 0$ , and  $y_1 = 1$ .
3.  $quotient = \lfloor i/j \rfloor$ .

4.  $remainder = i - (j * quotient)$ .
5.  $y = y_2 - (y_1 * quotient)$ .
6. Set  $i = j$ ,  $j = remainder$ ,  $y_2 = y_1$ , and  $y_1 = y$ .
7. If  $(j > 0)$ , then go to step 3.
8. If  $(i \neq 1)$ , then return an **ERROR** indication.
9. Return **SUCCESS** and  $y_2 \bmod a$ .

## C.2 Conversion Between Bit Strings and Integers

### C.2.1 Conversion of a Bit String to an Integer

An  $n$ -long sequence of bits  $\{x_1, \dots, x_n\}$  is converted to an integer by the rule

$$\{x_1, \dots, x_n\} \rightarrow (x_1 * 2^{n-1}) + (x_2 * 2^{n-2}) + \dots + (x_{n-1} * 2) + x_n.$$

Note that the first bit of a sequence corresponds to the most significant bit of the corresponding integer, and the last bit corresponds to the least significant bit.

#### Input:

1.  $b_1, b_2, \dots, b_n$  The bit string to be converted.

#### Output:

1.  $C$  The requested integer representation of the bit string.

#### Process:

1. Let  $(b_1, b_2, \dots, b_n)$  be the bits of  $b$  from leftmost to rightmost.
2.  $C = \sum_{i=1}^n 2^{(n-i)} b_i$
3. Return  $C$ .

In this Standard, the binary length of an integer  $C$  is defined as the smallest integer  $n$  satisfying  $C < 2^n$ .

### C.2.2 Conversion of an Integer to a Bit String

An integer  $x$  in the range  $0 \leq x < 2^n$  may be converted to an  $n$ -long sequence of bits by using its binary expansion as shown below:

$$x = (x_1 * 2^{n-1}) + (x_2 * 2^{n-2}) + \dots + (x_{n-1} * 2) + x_n \rightarrow \{x_1, \dots, x_n\}$$

Note that the first bit of a sequence corresponds to the most significant bit of the corresponding integer, and the last bit corresponds to the least significant bit.

**Input:**

1.  $C$  The non-negative integer to be converted.

**Output:**

1.  $b_1, b_2, \dots, b_n$  The bit string representation of the integer  $C$ .

**Process:**

1. Let  $(b_1, b_2, \dots, b_n)$  represent the bit string, where  $b_i = 0$  or  $1$ , and  $b_1$  is the most significant bit, while  $b_n$  is the least significant bit.
2. For any integer  $n$  that satisfies  $C < 2^n$ , the bits  $b_i$  **shall** satisfy:

$$C = \sum_{i=1}^n 2^{(n-i)} b_i$$

3. Return  $b_1, b_2, \dots, b_n$ .

In this Standard, the binary length of the integer  $C$  is defined as the smallest integer  $n$  that satisfies  $C < 2^n$ .

### C.3 Probabilistic Primality Tests

A probabilistic primality test may be required during the generation and validation of prime numbers. An approved robust probabilistic primality test **shall** be selected and used.

There are several probabilistic algorithms available. The Miller-Rabin probabilistic primality tests described in Appendices C.3.1 and C.3.2 are versions of a procedure due to M.O. Rabin, based in part on ideas of Gary L. Miller; one of these versions **shall** be used as the Miller-Rabin test discussed below. For more information, see [4]. For these tests, let **RBG** be an approved random bit generator.

There are several Lucas probabilistic primality tests available; the version provided in [5] is specified in Appendix C.3.3.

This Standard allows two alternatives for testing primality: either using several iterations of only the Miller-Rabin test, or using the iterated Miller-Rabin test, followed by a single Lucas test. The value of *iterations* (as used in Appendices C.3.1 and C.3.2) depends on the algorithm being used, the security strength, the error probability used, the length (in bits) of the candidate prime and

the type of tests to be performed. Tables C.1, C.2 and C.3 list the minimum number of *iterations* of the Miller-Rabin tests that **shall** be performed.

As stated in Appendix F, if the definition of the error probability that led to the values of the number of Miller-Rabin tests for  $p$  and  $q$  in Tables C.1, C.2 and C.3 is not conservative enough, the prescribed number of Miller-Rabin tests can be followed by a single Lucas test. Since there are no known non-prime values that pass the two test combination (i.e., the indicated number of rounds of the Miller-Rabin test with randomly selected bases, followed by one round of the Lucas test), the two test combination may provide additional assurance of primality over the use of only the Miller-Rabin test. For DSA, the two-test combination may provide better performance. However, the Lucas test is not required when testing the  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  values for primality when generating RSA primes. See Appendix F for further information.

**Table C.1. Minimum number of Miller-Rabin iterations for DSA**

Parameters	M-R Tests Only	M-R Tests when followed by One Lucas test
$p$ : 1024 bits $q$ : 160 bits Error probability = $2^{-80}$	For $p$ and $q$ : 40	For $p$ : 3 For $q$ : 19
$p$ : 2048 bits $q$ : 224 bits Error probability = $2^{-112}$	For $p$ and $q$ : 56	For $p$ : 3 For $q$ : 24
$p$ : 2048 bits $q$ : 256 bits Error probability = $2^{-112}$	For $p$ and $q$ : 56	For $p$ : 3 For $q$ : 27
$p$ : 3072 bits $q$ : 256 bits Error probability = $2^{-128}$	For $p$ and $q$ : 64	For $p$ : 2 For $q$ : 27



**Table C.2. Minimum number of rounds of M-R testing when generating primes for use in RSA Digital Signatures**

Parameters	M-R Tests Only
$p_1, p_2, q_1$ and $q_2 > 100$ bits $p$ and $q$ : 512 bits Error probability = $2^{-80}$	For $p_1, p_2, q_1$ and $q_2$ : 28 For $p$ and $q$ : 5
$p_1, p_2, q_1$ and $q_2 > 140$ bits $p$ and $q$ : 1024 bits Error probability = $2^{-112}$	For $p_1, p_2, q_1$ and $q_2$ : 38 For $p$ and $q$ : 5
$p_1, p_2, q_1$ and $q_2 > 170$ bits $p$ and $q$ : 1536 bits Error probability = $2^{-128}$	For $p_1, p_2, q_1$ and $q_2$ : 41 For $p$ and $q$ : 4

**Table C.3. Minimum number of rounds of M-R testing when generating primes for use in RSA Digital Signatures using an error probability of  $2^{-100}$**

Parameters	M-R Tests Only
$p_1, p_2, q_1$ and $q_2 > 100$ bits $p$ and $q$ : 512	For $p_1, p_2, q_1$ and $q_2$ : 38 For $p$ and $q$ : 7
$p_1, p_2, q_1$ and $q_2 > 140$ bits $p$ and $q$ : 1024 bits	For $p_1, p_2, q_1$ and $q_2$ : 32 For $p$ and $q$ : 4
$p_1, p_2, q_1$ and $q_2 > 170$ bits $p$ and $q$ : 1536 bits	For $p_1, p_2, q_1$ and $q_2$ : 27 For $p$ and $q$ : 3

### C.3.1 Miller-Rabin Probabilistic Primality Test

Let **RBG** be an approved random bit generator.

**Input:**

1.  $w$  The odd integer to be tested for primality. This will be either  $p$  or  $q$ , or one of the auxiliary primes  $p_1, p_2, q_1$  or  $q_2$ .

2. *iterations*                      The number of iterations of the test to be performed; the value **shall** be consistent with Table C.1, C.2 or C.3.

**Output:**

1. *status*                              The status returned from the validation procedure, where *status* is either **PROBABLY PRIME** or **COMPOSITE**.

**Process:**

1. Let  $a$  be the largest integer such that  $2^a$  divides  $w-1$ .
2.  $m = (w-1) / 2^a$ .
3.  $wlen = \text{len}(w)$ .
4. For  $i = 1$  to *iterations* do
  - 4.1 Obtain a string  $b$  of  $wlen$  bits from an RBG.  

Comment: Ensure that  $1 < b < w-1$ .
  - 4.2 If  $((b \leq 1) \text{ or } (b \geq w-1))$ , then go to step 4.1.
  - 4.3  $z = b^m \bmod w$ .
  - 4.4 If  $((z = 1) \text{ or } (z = w-1))$ , then go to step 4.7.
  - 4.5 For  $j = 1$  to  $a-1$  do.
    - 4.5.1  $z = z^2 \bmod w$ .
    - 4.5.2 If  $(z = w-1)$ , then go to step 4.7.
    - 4.5.3 If  $(z = 1)$ , then go to step 4.6.
  - 4.6 Return **COMPOSITE**.
  - 4.7 Continue.
 

Comment: Increment  $i$  for the do-loop in step 4.
5. Return **PROBABLY PRIME**.

### C.3.2 Enhanced Miller-Rabin Probabilistic Primality Test

This method provides additional information when an error is encountered that may be useful when generating or validating RSA moduli. Let **RBG** be an approved random bit generator.

**Input:**

1.  $w$                                       The odd integer to be tested for primality. This will be either  $p$  or  $q$ , or one of the auxiliary primes  $p_1, p_2, q_1$  or  $q_2$ .

2. *iterations* The number of iterations of the test to be performed; the value **shall** be consistent with Table C.1, C.2 or C.3.

**Output:**

1. *status* The status returned from the validation procedure, where *status* is either **PROBABLY PRIME**, **PROVABLY COMPOSITE WITH FACTOR** (returned with the factor), and **PROVABLY COMPOSITE AND NOT A POWER OF A PRIME**.

**Process:**

1. Let  $a$  be the largest integer such that  $2^a$  divides  $w-1$ .
2.  $m = (w-1) / 2^a$ .
3.  $wlen = \mathbf{len}(w)$ .
4. For  $i = 1$  to *iterations* do
  - 4.1 Obtain a string  $b$  of  $wlen$  bits from an RBG.  

Comment: Ensure that  $1 < b < w-1$ .
  - 4.2 If  $((b \leq 1) \text{ or } (b \geq w-1))$ , then go to step 4.1.
  - 4.3  $g = \mathbf{GCD}(b, w)$ .
  - 4.4 If  $(g > 1)$ , then return **PROVABLY COMPOSITE WITH FACTOR** and the value of  $g$ .
  - 4.5  $z = b^m \bmod w$ .
  - 4.6 If  $((z = 1) \text{ or } (z = w - 1))$ , then go to step 4.15.
  - 4.7 For  $j = 1$  to  $a - 1$  do.
    - 4.7.1  $x = z$ . 

Comment:  $x \neq 1$  and  $x \neq w-1$ .
    - 4.7.2  $z = x^2 \bmod w$ .
    - 4.7.3 If  $(z = w-1)$ , then go to step 4.15.
    - 4.7.4 If  $(z = 1)$ , then go to step 4.12.
  - 4.8  $x = z$ . 

Comment:  $x = b^{(w-1)/2} \bmod w$  and  $x \neq w-1$ .
  - 4.9  $z = x^2 \bmod w$ .
  - 4.10 If  $(z = 1)$ , then go to step 4.12.
  - 4.11  $x = z$ . 

Comment:  $x = b^{(w-1)} \bmod w$  and  $x \neq 1$ .

- 4.12  $g = \mathbf{GCD}(x-1, w)$ .
- 4.13 If  $(g > 1)$ , then return **PROVABLY COMPOSITE WITH FACTOR** and the value of  $g$ .
- 4.14 Return **PROVABLY COMPOSITE AND NOT A POWER OF A PRIME**.
- 4.15 Continue. Comment: Increment  $i$  for the do-loop in step 4.
5. Return **PROBABLY PRIME**.

### C.3.3 (General) Lucas Probabilistic Primality Test

The following process or its equivalent **shall** be used as the Lucas test.

**Input:**

$C$  The candidate odd integer to be tested for primality.

**Output:**

*status* Where *status* is either **PROBABLY PRIME** or **COMPOSITE**.

**Process:**

1. Test whether  $C$  is a perfect square (see Appendix C.4). If so, return (**COMPOSITE**).
2. Find the first  $D$  in the sequence  $\{5, -7, 9, -11, 13, -15, 17, \dots\}$  for which the Jacobi symbol  $\left(\frac{D}{C}\right) = -1$ . See Appendix C.5 for an approved method to compute the Jacobi Symbol. If  $\left(\frac{D}{C}\right) = 0$  for any  $D$  in the sequence, return (**COMPOSITE**).
3.  $K = C+1$ .
4. Let  $K_r K_{r-1} \dots K_0$  be the binary expansion of  $K$ , with  $K_r = 1$ .
5. Set  $U_r = 1$  and  $V_r = 1$ .
6. For  $i = r-1$  to 0, do
  - 6.1  $U_{temp} = U_{i+1} V_{i+1} \bmod C$ .
  - 6.2  $V_{temp} = \frac{V_{i+1}^2 + D U_{i+1}^2}{2} \bmod C$ .
  - 6.3 If  $(K_i = 1)$ , then Comment: If  $K_i = 1$ , then do steps 6.3.1 and 6.3.2; otherwise, do steps 6.3.3 and 6.3.4.
    - 6.3.1  $U_i = \frac{U_{temp} + V_{temp}}{2} \bmod C$ .

$$6.3.2 \quad V_i = \frac{V_{temp} + DU_{temp}}{2} \bmod C.$$

Else

$$6.3.3 \quad U_i = U_{temp}.$$

$$6.3.4 \quad V_i = V_{temp}.$$

7. If  $(U_0 = 0)$ , then return (**PROBABLY PRIME**). Otherwise, return (**COMPOSITE**).

Steps 6.2, 6.3.1 and 6.3.2 contain expressions of the form  $A/2 \bmod C$ , where  $A$  is an integer, and  $C$  is an odd integer. If  $A/2$  is not an integer (i.e.,  $A$  is odd), then  $A/2 \bmod C$  may be calculated as  $(A+C)/2 \bmod C$ . Alternatively,  $A/2 \bmod C = A \cdot (C+1)/2 \bmod C$ , for any integer  $A$ , without regard to  $A$  being odd or even.

#### C.4 Checking for a Perfect Square

The following algorithm may be used to determine whether an  $n$ -bit positive integer  $C$  is a perfect square:

**Input:**

$C$  The integer to be checked.

**Output:**

*status* Where *status* is either **PERFECT SQUARE** or **NOT A PERFECT SQUARE**.

**Process:**

1. Set  $n$ , such that  $2^n > C \geq 2^{(n-1)}$ .
2.  $m = \lceil n/2 \rceil$ .
3.  $i = 0$ .
4. Select  $X_0$ , such that  $2^m > X_0 \geq 2^{(m-1)}$ .
5. Repeat
  - 5.1  $i = i + 1$ .
  - 5.2  $X_i = ((X_{i-1})^2 + C)/(2X_{i-1})$ .
 Until  $(X_i)^2 < 2^m + C$ .
6. If  $C = \lfloor X_i \rfloor^2$ , then
 

*status* = **PERFECT SQUARE**.

Else

*status* = **NOT A PERFECT SQUARE**.

7. **Return status.**

**Notes:**

1. By starting with  $X_0 > (1/2) \mathbf{Sqrt}(C)$ ,  $|X_0 - \mathbf{Sqrt}(C)|$  is guaranteed to be less than  $X_0$ . This inequality is maintained in step 5; i.e.,  $|X_i - \mathbf{Sqrt}(C)| < X_i$  for all  $i$ .
2. For  $i \geq 1$ ,  $0 \leq X_i - \mathbf{Sqrt}(C) = (X_{i-1} - \mathbf{Sqrt}(C))^2 / (2 X_{i-1}) < X_0 / 2^i$ .

In particular,  $0 \leq X_m - \mathbf{Sqrt}(C) < 1$ . If  $\mathbf{Sqrt}(C)$  were an integer, then it would be equal to the floor of  $X_m$ .

3. In general, the inequality  $X_i - \mathbf{Sqrt}(C) < 1$  will occur for values of  $i$  that are much less than  $m$ . To detect this, the fact that  $2^{(m-1)} \leq \mathbf{Sqrt}(C) < X_i$  for all  $i \geq 1$  can be used,

$$\begin{aligned} X_i - \mathbf{Sqrt}(C) &= ((X_i)^2 - C) / (X_i + \mathbf{Sqrt}(C)) \\ &\leq ((X_i)^2 - C) / (2 \mathbf{Sqrt}(C)) \\ &\leq ((X_i)^2 - C) / (2^m) \end{aligned}$$

Thus, the condition  $(X_i)^2 < 2^m + C$  implies that  $X_i - \mathbf{Sqrt}(C) < 1$ .

## C.5 Jacobi Symbol Algorithm

This routine computes the Jacobi symbol  $\left(\frac{a}{n}\right)$ .

**Jacobi():**

**Input:**

- $a$  Any integer. For this Standard, the initial value is in the sequence  $\{5, -7, 9, -11, 13, -15, 17, \dots\}$ , as determined by Appendix C.3.3.
- $n$  Any integer. For this Standard, the initial value is the candidate being tested, as determined by Appendix C.3.3.

**Output:**

*result* The calculated Jacobi symbol.

**Process:**

1.  $a = a \bmod n$ . Comment:  $a$  will be in the range  $0 \leq a < n$ .
2. If  $a = 1$ , or  $n = 1$ , then return (1).
3. If  $a = 0$ , then return (0).
4. Define  $e$  and  $a_1$  such that  $a = 2^e a_1$ , where  $a_1$  is odd.

5. If  $e$  is even, then  $s = 1$ .  
     Else if  $((n \equiv 1 \pmod{8}) \text{ or } (n \equiv 7 \pmod{8}))$ , then  $s = 1$ .  
     Else if  $((n \equiv 3 \pmod{8}) \text{ or } (n \equiv 5 \pmod{8}))$ , then  $s = -1$ .
6. If  $((n \equiv 3 \pmod{4}) \text{ and } (a_1 \equiv 3 \pmod{4}))$ , then  $s = -s$ .
7.  $n_1 = n \bmod a_1$ .
8. Return  $(s * \text{Jacobi}(n_1, a_1))$ .      Comment: Call this routine recursively.

Example: Compute the Jacobi symbol for  $a = 5$  and  $n = 3439601197$ :

1.  $n$  is not 1, and  $a$  is not 1, so proceed to Step 2.
2.  $a$  is not 0, so proceed to Step 3.
3.  $5 = 2^0 * 5$ , so  $e = 0$ , and  $a_1 = 5$ .
4.  $e$  is even, so  $s = 1$ .
5.  $a_1$  is not congruent to 3 mod 4, so do not change  $s$ .
6.  $n_1 = 2 = n \bmod 5$ .
7. Compute and return  $(1 * \text{Jacobi}(2, 5))$ . This calls Jacobi recursively. Compute the Jacobi symbol for  $a = 2$  and  $n = 5$ :
  - 7.1  $n$  is not 1, and  $a$  is not 1, so proceed to Step 7.2.
  - 7.2  $a$  is not 0, so proceed to Step 7.3.
  - 7.3  $2 = 2^1 * 1$ , so  $e = 1$ , and  $a_1 = 1$ .
  - 7.4  $e$  is odd, and  $n \equiv 5 \pmod{8}$ , so set  $s = -1$ .
  - 7.5  $n$  is not 3 mod 4, and  $a_1$  is not 3 mod 4, so proceed to step 7.6.
  - 7.6  $n_1 = 0 = n \bmod 1$ .
  - 7.7 Return  $(-1 * \text{Jacobi}(0, 1) = -1)$ . This calls Jacobi recursively. Compute the Jacobi symbol for  $a = 0$  and  $n = 1$ :
    - 7.7.1  $n = 1$ , so return 1.

Thus,  $\text{Jacobi}(0, 1) = 1$ , so  $\text{Jacobi}(2, 5) = -1 * (1) = -1$ , and  $\text{Jacobi}(5, 3439601197) = 1 * (-1) = -1$ .

## C.6 Shawe-Taylor Random\_Prime Routine

This routine is recursive and may be used to construct a provable prime number using a hash function.

Let **Hash**( ) be the selected hash function, and let *outlen* be the bit length of the hash function output block. The following process or its equivalent **shall** be used to generate a prime number for this constructive method.

**ST\_Random\_Prime ( ):**

**Input:**

1. *length*                      The length of the prime to be generated.
2. *input\_seed*                The seed to be used for the generation of the requested prime.

**Output:**

1. *status*                      The status returned from the generation routine, where *status* is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned, then zeros are returned as the other output values.
2. *prime*                        The requested prime.
3. *prime\_seed*                A seed determined during generation.
4. *prime\_gen\_counter*      (Optional) A counter determined during the generation of the prime.

**Process:**

1. If (*length* < 2), then return (**FAILURE**, 0, 0 { , 0 }).
2. If (*length* ≥ 33), then go to step 14.
3. *prime\_seed* = *input\_seed*.
4. *prime\_gen\_counter* = 0.

Comment: Generate a pseudorandom integer *c* of *length* bits.

5.  $c = \text{Hash}(\text{prime\_seed}) \oplus \text{Hash}(\text{prime\_seed} + 1)$ .
6.  $c = 2^{\text{length} - 1} + (c \bmod 2^{\text{length} - 1})$ .
7.  $c = (2 * \lfloor c / 2 \rfloor) + 1$ .

Comment: Set *prime* to the least odd integer greater than or equal to *c*.

8. *prime\_gen\_counter* = *prime\_gen\_counter* + 1.
9. *prime\_seed* = *prime\_seed* + 2.



10. Perform a deterministic primality test on  $c$ . For example, since  $c$  is small, its primality can be tested by trial division. See Appendix C.7.

11. If ( $c$  is a prime number), then

11.1  $prime = c$ .

11.2 Return (**SUCCESS**,  $prime$ ,  $prime\_seed$  {,  $prime\_gen\_counter$ }).

12. If ( $prime\_gen\_counter > (4 * length)$ ), then return (**FAILURE**, 0, 0 {, 0}).

13. Go to step 5.

14. ( $status, c_0, prime\_seed, prime\_gen\_counter$ ) = (**ST\_Random\_Prime** ( $(\lceil length / 2 \rceil + 1)$ ,  $input\_seed$ )).

15. If **FAILURE** is returned, return (**FAILURE**, 0, 0 {, 0}).

16.  $iterations = \lceil length / outlen \rceil - 1$ .

17.  $old\_counter = prime\_gen\_counter$ .

Comment: Generate a pseudorandom integer  $x$  in the interval  $[2^{length-1}, 2^{length}]$ .

18.  $x = 0$ .

19. For  $i = 0$  to  $iterations$  do

$x = x + (\text{Hash}(prime\_seed + i) * 2^{i * outlen})$ .

20.  $prime\_seed = prime\_seed + iterations + 1$ .

21.  $x = 2^{length-1} + (x \bmod 2^{length-1})$ .

Comment: Generate a candidate prime  $c$  in the interval  $[2^{length-1}, 2^{length}]$ .

22.  $t = \lceil x / (2c_0) \rceil$ .

23. If  $(2tc_0 + 1 > 2^{length})$ , then  $t = \lceil 2^{length-1} / (2c_0) \rceil$ .

24.  $c = 2tc_0 + 1$ .

25.  $prime\_gen\_counter = prime\_gen\_counter + 1$ .

Comment: Test the candidate prime  $c$  for primality; first pick an integer  $a$  between 2 and  $c - 2$ .

26.  $a = 0$ .

27. For  $i = 0$  to  $iterations$  do

$a = a + (\text{Hash}(prime\_seed + i) * 2^{i * outlen})$ .

28.  $prime\_seed = prime\_seed + iterations + 1$ .
29.  $a = 2 + (a \bmod (c - 3))$ .
30.  $z = a^{2^t} \bmod c$ .
31. If  $((1 = \mathbf{GCD}(z - 1, c)) \text{ and } (1 = z^{c_0} \bmod c))$ , then
  - 31.1  $prime = c$ .
  - 31.2 Return (**SUCCESS**,  $prime$ ,  $prime\_seed$  {,  $prime\_gen\_counter$ }).
32. If  $(prime\_gen\_counter \geq ((4 * length) + old\_counter))$ , then return (**FAILURE**, 0, 0 {, 0}).
33.  $t = t + 1$ .
34. Go to step 23.

## C.7 Trial Division

An integer is proven to be prime by showing that it has no prime factors less than or equal to its square root. This procedure is not recommended for testing any integers longer than 10 digits.

To prove that  $c$  is prime:

1. Prepare a table of primes less than  $\sqrt{c}$ . This can be done by applying the sieve procedure in Appendix C.8.
2. Divide  $c$  by every prime in the table. If  $c$  is divisible by one of the primes, then declare that  $c$  is composite and exit. If convenient,  $c$  may be divided by composite numbers. For example, rather than preparing a table of primes, it might be more convenient to divide by all integers except those divisible by 3 or 5.
3. Otherwise, declare that  $c$  is prime and exit.

## C.8 Sieve Procedure

A *sieve procedure* is described as follows: Given a sequence of integers  $Y_0, Y_0 + 1, \dots, Y_0 + J$ , a sieve will identify the integers in the sequence that are divisible by primes up to some selected limit.

Note that the definitions of the mathematical symbols in this process (e.g.,  $h, L, M, p$ ) are internal to this process only, and should not be confused with their use elsewhere in this Standard.

Start by selecting a *factor base* of all the primes  $p_j$ , from 2 up to some selected limit  $L$ . The value of  $L$  is arbitrary and may be determined by computer limitations. A good, typical value of  $L$  would be anywhere from  $10^3$  to  $10^5$ .

1. Compute  $S_j = Y_0 \bmod p_j$  for all  $p_j$  in the factor base.
2. Initialize an array of length  $J + 1$  to zero.
3. Starting at  $Y_0 - S_j + p_j$ , let every  $p_j^{\text{th}}$  element of the array be set to 1. Do this for the entire length of the array and for every  $j$ .
4. When finished, every location in the array that has the value 1 is divisible by some small prime, and is therefore a composite.

The array can be either a bit array for compactness when memory is small, or a byte array for speed when memory is readily available. There is no need to sieve the entire sieve interval at once. The array can be partitioned into suitably small pieces, sieving each piece before going on to the next piece. When finished, every location with the value 0 is a candidate for prime testing.

The amount of work for this procedure is approximately  $M \log \log L$ , where  $M$  is the length of the sieve interval; this is a very efficient procedure for removing composite candidates for primality testing. If  $L = 10^5$ , the sieve will remove about 96% of all composites.

In some cases, rather than having a set of consecutive integers to sieve, the set of integers to be tested consists of integers lying in an arithmetic progression  $Y_0, Y_0 + h, Y_0 + 2h, \dots, Y_0 + Jh$ , where  $h$  is large and not divisible by any primes in the factor base.

1. Select a factor base and initialize an array of length  $J + 1$  to 0.
2. Compute  $S_j = Y_0 \bmod p_j$  for all  $p_j$  in the factor base.
3. Compute  $T_j = h \bmod p_j$  and  $r = -S_j T_j^{-1} \bmod p_j$ .
4. Starting at  $Y_0 + r$ , let every  $p_j^{\text{th}}$  element of the array be set to 1. Do this for the entire length of the array and for every  $j$ . Note that the position  $Y_0 + r$  in the array actually denotes the number  $Y_0 + rh$ .
5. When finished, every location in the array that has the value 1 is divisible by some small prime and is therefore composite.

Note: The prime “2” takes the longest amount of time ( $M/2$ ) to sieve, since it touches the most locations in the sieve array. An easy optimization is to combine the initialization of the sieve array with the sieving of the prime “2”. It is also possible to sieve the prime “3” during initialization. These optimizations can save about 1/3 of the total sieve time.

### C.9 Compute a Probable Prime Factor Based on Auxiliary Primes

This routine constructs a probable prime (a candidate for  $p$  or  $q$ ) using two auxiliary prime numbers and the Chinese Remainder Theorem (CRT).

**Input:**

$r_1$ and $r_2$	Two odd prime numbers satisfying $\log_2(r_1 r_2) \leq (n \text{len}/2) - \log_2(n \text{len}/2) - 6$ .
-----------------	--

<i>nlen</i>	The desired length of <i>n</i> , the RSA modulus.
<i>e</i>	The public verification exponent.
<i>security_strength</i>	The minimum security strength required for random number generation.

**Output:**

<i>status</i>	The status returned from the generation procedure, where <i>status</i> is either <b>SUCCESS</b> or <b>FAILURE</b> . If <b>FAILURE</b> is returned, then zeros are returned as the other output values.
<i>private_prime_factor</i>	The prime factor of <i>n</i> .
<i>X</i>	The random number used during the generation of the <i>private_prime_factor</i> .

**Process:**

1. If  $(\text{GCD}(2r_1, r_2) \neq 1)$ , then return (**FAILURE**, 0, 0).
2.  $R = ((r_2^{-1} \bmod 2r_1) * r_2) - (((2r_1)^{-1} \bmod r_2) * 2r_1)$ .  

Comment: Apply the CRT, so that  $R \equiv 1 \pmod{2r_1}$  and  $R \equiv -1 \pmod{r_2}$ .
3. Generate a random number *X* using an approved random number generator that supports the *security\_strength*, such that  $(\sqrt{2})(2^{nlen/2-1}) \leq X \leq (2^{nlen/2} - 1)$ .
4.  $Y = X + ((R - X) \bmod 2r_1 r_2)$ .  

Comment: *Y* is the first odd integer  $\geq X$ , such that  $r_1$  is a prime factor of  $Y-1$ , and  $r_2$  is a prime factor of  $Y+1$ .

Comment: Determine the requested prime number by constructing candidates from a sequence and performing primality tests.
5.  $i = 0$ .
6. If  $(Y \geq 2^{nlen/2})$ , then go to step 3.
7. If  $(\text{GCD}(Y-1, e) = 1)$ , then
  - 7.1 Check the primality of *Y* as specified in Appendix C.3. If **PROBABLY PRIME** is **not** returned, go to step 8.
  - 7.2  $\text{private\_prime\_factor} = Y$ .
  - 7.3 Return (**SUCCESS**, *private\_prime\_factor*, *X*).
8.  $i = i + 1$ .

9. If  $(i \geq 5(nlen/2))$ , then return (**FAILURE**, 0, 0).
10.  $Y = Y + (2r_1r_2)$ .
11. Go to step 6.

### C.10 Construct a Provable Prime (possibly with Conditions), Based on Contemporaneously Constructed Auxiliary Provable Primes

The following process (or its equivalent) **shall** be used to generate an  $L$ -bit provable prime  $p$  (a candidate for one of the prime factors of an RSA modulus). Note that the use of  $p$  in this specification is used generically; both RSA prime factors  $p$  and  $q$  may be generated using this method.

If a so-called “strong prime” is required, this process can generate primes  $p_1$  and  $p_2$  (of specified bit-lengths  $N_1$  and  $N_2$ ) that divide  $p-1$  and  $p+1$ , respectively. The resulting prime  $p$  will satisfy the conditions traditionally required of a strong prime, provided that the requested bit-lengths for  $p_1$  and  $p_2$  have appropriate sizes.

Regardless of the bit-lengths selected for  $p_1$  and  $p_2$ , the quantity  $p-1$  will have a prime divisor  $p_0$  whose bit-length is slightly more than half that of  $p$ . In addition, the quantity  $p_0-1$  will have a prime divisor whose bit-length is slightly more than half that of  $p_0$ .

This algorithm requires that  $N_1 + N_2 \leq L - \lceil L/2 \rceil - 4$ . Values for  $N_1$  and  $N_2$  **should** be chosen such that  $N_1 + N_2 \leq (L/2) - \log_2(L) - 7$ , to ensure that the algorithm can generate as many as  $5L$  distinct candidates for  $p$ .

Let **Hash** be the selected hash function to be used, and let *outlen* be the bit length of the hash function output block.

#### Provable\_Prime\_Construction():

##### Input:

1.  $L$  A positive integer equal to the requested bit-length for  $p$ . Note that acceptable values for  $L = nlen/2$  are computed as specified in Appendix B.3.1, criteria 2(b) and (c), with  $nlen$  assuming a value specified in Table B.1.
2.  $N_1$  A positive integer equal to the requested bit-length for  $p_1$ . If  $N_1 \geq 2$ , then  $p_1$  is an odd prime of  $N_1$  bits; otherwise,  $p_1 = 1$ . Acceptable values for  $N_1 \geq 2$  are provided in Table B.1
3.  $N_2$  A positive integer equal to the requested bit-length for  $p_2$ . If  $N_2 \geq 2$ , then  $p_2$  is an odd prime of  $N_2$  bits; otherwise,  $p_2 = 1$ . Acceptable values for  $N_2 \geq 2$  are provided in Table B.1
4. *firstseed* A bit string equal to the first seed to be used.

5.  $e$  The public verification exponent.

**Output:**

1.  $status$  The status returned from the generation procedure, where  $status$  is either **SUCCESS** or **FAILURE**. If **FAILURE** is returned, then zeros are returned as the other output values.
2.  $p, p_1, p_2$  The required prime  $p$ , along with  $p_1$  and  $p_2$  having the property that  $p_1$  divides  $p-1$  and  $p_2$  divides  $p+1$ .
3.  $pseed$  A seed determined during generation.

**Process:**

1. If  $L, N_1$ , and  $N_2$  are not acceptable, then, return (**FAILURE**, 0, 0, 0, 0).  

Comment: Generate  $p_1$  and  $p_2$ , as well as the prime  $p_0$ .
2. If  $N_1 = 1$ , then
  - 2.1  $p_1 = 1$ .
  - 2.2  $p_2seed = firstseed$ .
3. If  $N_1 \geq 2$ , then
  - 3.1 Using  $N_1$  as the length and  $firstseed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $p_1$  and  $p_2seed$ .
  - 3.2 If **FAILURE** is returned, then return (**FAILURE**, 0, 0, 0, 0).
4. If  $N_2 = 1$ , then
  - 4.1  $p_2 = 1$ .
  - 4.2  $p_0seed = p_2seed$ .
5. If  $N_2 \geq 2$ , then
  - 5.1 Using  $N_2$  as the length and  $p_2seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $p_2$  and  $p_0seed$ .
  - 5.2 If **FAILURE** is returned, then return (**FAILURE**, 0, 0, 0, 0).
6. Using  $\lceil L / 2 \rceil + 1$  as the length and  $p_0seed$  as the  $input\_seed$ , use the random prime generation routine in Appendix C.6 to obtain  $p_0$  and  $pseed$ . If **FAILURE** is returned, then return (**FAILURE**, 0, 0, 0, 0).

Comment: Generate a (strong) prime  $p$  in the interval  $[(\sqrt{2})(2^{L-1}), 2^L - 1]$ .

7.  $iterations = \lceil L / outlen \rceil - 1$ .

8.  $p_{gen\_counter} = 0$ .

Comment: Generate pseudo-random  $x$  in the interval  $[(\sqrt{2})(2^{L-1})-1, 2^L-1]$ .

9.  $x = 0$ .

10. For  $i = 0$  to  $iterations$  do

$$x = x + (\mathbf{Hash}(p_{seed} + i)) * 2^{i * outlen}.$$

11.  $p_{seed} = p_{seed} + iterations + 1$ .

12.  $x = \lfloor (\sqrt{2})(2^{L-1}) \rfloor + (x \bmod (2^L - \lfloor (\sqrt{2})(2^{L-1}) \rfloor))$ .

Comment: Generate a candidate for the prime  $p$ .

13. If  $(\mathbf{GCD}(p_0 p_1, p_2) \neq 1)$ , then return **(FAILURE, 0, 0, 0, 0)**.

14. Compute  $y$  in the interval  $[1, p_2]$  such that  $0 = (y p_0 p_1 - 1) \bmod p_2$ .

15.  $t = \lceil ((2 y p_0 p_1) + x) / (2 p_0 p_1 p_2) \rceil$ .

16. If  $((2(t p_2 - y) p_0 p_1 + 1) > 2^L)$ , then

$$t = \lceil ((2 y p_0 p_1) + \lfloor (\sqrt{2})(2^{L-1}) \rfloor) / (2 p_0 p_1 p_2) \rceil.$$

Comment:  $p$  satisfies

$$0 = (p-1) \bmod (2 p_0 p_1) \text{ and}$$

$$0 = (p+1) \bmod p_2.$$

17.  $p = 2(t p_2 - y) p_0 p_1 + 1$ .

18.  $p_{gen\_counter} = p_{gen\_counter} + 1$ .

19. If  $(\mathbf{GCD}(p-1, e) = 1)$ , then

Comment: Choose an integer  $a$  in the interval  $[2, p-2]$ .

19.1  $a = 0$

19.2 For  $i = 0$  to  $iterations$  do

$$a = a + (\mathbf{Hash}(p_{seed} + i)) * 2^{i * outlen}.$$

19.3  $p_{seed} = p_{seed} + iterations + 1$ .

19.4  $a = 2 + (a \bmod (p-3))$ .

Comment: Test  $p$  for primality:

19.5  $z = a^{2(t p_2 - y) p_1} \bmod p$ .

- 19.6 If  $((1 = \mathbf{GCD}(z-1, p))$  and  $(1 = (z^{p_0} \bmod p))$ , then return (**SUCCESS**,  $p$ ,  $p_1$ ,  $p_2$ ,  $pseed$ ).
20. If  $(p_{gen\_counter} \geq 5L)$ , then return (**FAILURE**, 0, 0, 0, 0).
21.  $t = t + 1$ .
22. Go to step 16.



## Appendix D: Recommended Elliptic Curves for Federal Government Use

This collection of elliptic curves is recommended for Federal government use and contains choices for the private key length and underlying fields. These curves were generated using SHA-1 and the method given in the ANS X9.62 and IEEE Standard 1363-2000 standards. This appendix describes the process that was used. Note that these curves are the same as those included in the previous version of this Standard.

### D.1 NIST Recommended Elliptic Curves

#### D.1.1 Choices

##### D.1.1.1 Choice of Key Lengths

The principal parameters for elliptic curve cryptography are the elliptic curve  $E$  and a designated point  $G$  on  $E$  called the *base point*. The base point has order  $n$ , which is a large prime. The number of points on the curve is  $hn$  for some integer  $h$  (the *cofactor*), which is not divisible by  $n$ . For efficiency reasons, it is desirable to have the cofactor be as small as possible.

All of the curves given below have cofactors 1, 2, or 4. As a result, the private and public keys for a curve are approximately the same length.

##### D.1.1.2 Choice of Underlying Fields

For each key length, two kinds of fields are provided.

- A *prime field* is the field  $GF(p)$ , which contains a prime number  $p$  of elements. The elements of this field are the integers modulo  $p$ , and the field arithmetic is implemented in terms of the arithmetic of integers modulo  $p$ .
- A *binary field* is the field  $GF(2^m)$ , which contains  $2^m$  elements for some  $m$  (called the *degree* of the field). The elements of this field are the bit strings of length  $m$ , and the field arithmetic is implemented in terms of operations on the bits.

The security strengths for five ranges of the bit length of  $n$  is provided in SP 800-57. For the field  $GF(p)$ , the security strength is dependent on the length of the binary expansion of  $p$ . For the field  $GF(2^m)$ , the security strength is dependent on the value of  $m$ . Table E-1 provides the bit lengths of the various underlying fields of the curves provided in this appendix. Column 1 lists the ranges for the bit length of  $n$  (also see Table 1 in Section 6.1.1). Column 2 identifies the value of  $p$  used for the curves over prime fields, where  $\text{len}(p)$  is the length of the binary expansion of the integer  $p$ . Column 3 provides the value of  $m$  for the curves over binary fields.

**Table D-1: Bit Lengths of the Underlying Fields of the Recommended Curves**

Bit Length of $n$	Prime Field	Binary Field
161 – 223	$\text{len}(p) = 192$	$m = 163$
224 – 255	$\text{len}(p) = 224$	$m = 233$
256 – 383	$\text{len}(p) = 256$	$m = 283$
384 – 511	$\text{len}(p) = 384$	$m = 409$
$\geq 512$	$\text{len}(p) = 521$	$m = 571$

**D.1.1.3 Choice of Basis for Binary Fields**

To describe the arithmetic of a binary field, it is first necessary to specify how a bit string is to be interpreted. This is referred to as choosing a *basis* for the field. There are two common types of bases: a *polynomial basis* and a *normal basis*.

- A polynomial basis is specified by an irreducible polynomial modulo 2, called the *field polynomial*. The bit string  $(a_{m-1} \dots a_2 a_1 a_0)$  is taken to represent the polynomial

$$a_{m-1} t^{m-1} + \dots + a_2 t^2 + a_1 t + a_0$$

over  $GF(2)$ . The field arithmetic is implemented as polynomial arithmetic modulo  $p(t)$ , where  $p(t)$  is the field polynomial.

- A normal basis is specified by an element  $\theta$  of a particular kind. The bit string  $(a_0 a_1 a_2 \dots a_{m-1})$  is taken to represent the element

$$a_0 \theta + a_1 \theta^2 + a_2 \theta^{2^2} + \dots + a_{m-1} \theta^{2^{m-1}}.$$

Normal basis field arithmetic is not easy to describe or efficient to implement in general, except for a special class called *Type T low-complexity* normal bases. For a given field degree  $m$ , the choice of  $T$  specifies the basis and the field arithmetic (see Appendix D.3).

There are many polynomial bases and normal bases from which to choose. The following procedures are commonly used to select a basis representation.

- *Polynomial Basis*: If an irreducible *trinomial*  $t^m + t^k + 1$  exists over  $GF(2)$ , then the field polynomial  $p(t)$  is chosen to be the irreducible trinomial with the lowest-degree middle term  $t^k$ . If no irreducible trinomial exists, then a *pentanomial*  $t^m + t^a + t^b + t^c + 1$  is selected. The particular pentanomial chosen has the following properties: the second term  $t^a$  has the lowest degree  $m$ ; the third term  $t^b$  has the lowest degree among all irreducible pentanomials of degree  $m$  and second term  $t^a$ ; and the fourth term  $t^c$  has the lowest degree among all irreducible pentanomials of degree  $m$ , second term  $t^a$ , and third term  $t^b$ .

- *Normal Basis*: Choose the Type T low-complexity normal basis with the smallest  $T$ .

For each binary field, the parameters are given for the above basis representations.

#### D.1.1.4 Choice of Curves

Two kinds of curves are given:

- *Pseudo-random* curves are those whose coefficients are generated from the output of a seeded cryptographic hash function. If the domain parameter seed value is given along with the coefficients, it can be easily verified that the coefficients were generated by that method.
- *Special curves* are those whose coefficients and underlying field have been selected to optimize the efficiency of the elliptic curve operations.

For each curve size range, the following curves are given:

- A pseudo-random curve over  $GF(p)$ .
- A pseudo-random curve over  $GF(2^m)$ .
- A special curve over  $GF(2^m)$  called a *Koblitz curve* or *anomalous binary curve*.

The pseudo-random curves were generated as specified in ANS X9.62 using SHA-1.

#### D.1.1.5 Choice of Base Points

Any point of order  $n$  can serve as the base point. Each curve is supplied with a sample base point  $G = (G_x, G_y)$ . Users may want to generate their own base points to ensure cryptographic separation of networks. See ANS X9.62 or IEEE Standard 1363-2000.

### D.1.2 Curves over Prime Fields

For each prime  $p$ , a pseudo-random curve

$$E: y^2 \equiv x^3 - 3x + b \pmod{p}$$

of prime order  $n$  is listed<sup>4</sup>. (Thus, for these curves, the cofactor is always  $h = 1$ .) The following parameters are given:

- The prime modulus  $p$
- The order  $n$
- The 160-bit input seed *SEED* to the SHA-1 based algorithm (i.e., the domain parameter seed)
- The output  $c$  of the SHA-1 based algorithm

---

<sup>4</sup> The selection  $a \equiv -3$  for the coefficient of  $x$  was made for reasons of efficiency; see IEEE Std 1363-2000.

- The coefficient  $b$  (satisfying  $b^2 c \equiv -27 \pmod{p}$ )
- The base point  $x$  coordinate  $G_x$
- The base point  $y$  coordinate  $G_y$

The integers  $p$  and  $n$  are given in decimal form; bit strings and field elements are given in hexadecimal.

#### D.1.2.1 Curve P-192

$p =$  6277101735386680763835789423207666416083908700390324961279  
 $n =$  6277101735386680763835789423176059013767194773182842284081  
 $SEED =$  3045ae6f c8422f64 ed579528 d38120ea e12196d5  
 $c =$  3099d2bb bfcbb2538 542dcd5f b078b6ef 5f3d6fe2 c745de65  
 $b =$  64210519 e59c80e7 0fa7e9ab 72243049 feb8deec c146b9b1  
 $G_x =$  188da80e b03090f6 7cbf20eb 43a18800 f4ff0afd 82ff1012  
 $G_y =$  07192b95 ffc8da78 631011ed 6b24cdd5 73f977a1 1e794811

#### D.1.2.2 Curve P-224

$p =$  2695994666715063979466701508701963067355791626002630814351  
 0066298881  
 $n =$  2695994666715063979466701508701962594045780771442439172168  
 2722368061  
 $SEED =$  bd713447 99d5c7fc dc45b59f a3b9ab8f 6a948bc5  
 $c =$  5b056c7e 11dd68f4 0469ee7f 3c7a7d74 f7d12111 6506d031  
 218291fb  
 $b =$  b4050a85 0c04b3ab f5413256 5044b0b7 d7bfd8ba 270b3943  
 2355ffb4  
 $G_x =$  b70e0cbd 6bb4bf7f 321390b9 4a03c1d3 56c21122 343280d6  
 115c1d21  
 $G_y =$  bd376388 b5f723fb 4c22dfe6 cd4375a0 5a074764 44d58199  
 85007e34

### D.1.2.3 Curve P-256

$p =$  1157920892103562487626974469494075735300861434152903141955  
33631308867097853951

$n =$  115792089210356248762697446949407573529996955224135760342  
422259061068512044369

$SEED =$  c49d3608 86e70493 6a6678e1 139d26b7 819f7e90

$c =$  7efba166 2985be94 03cb055c 75d4f7e0 ce8d84a9 c5114abc  
af317768 0104fa0d

$b =$  5ac635d8 aa3a93e7 b3ebbd55 769886bc 651d06b0 cc53b0f6  
3bce3c3e 27d2604b

$G_x =$  6b17d1f2 e12c4247 f8bce6e5 63a440f2 77037d81 2deb33a0  
f4a13945 d898c296

$G_y =$  4fe342e2 fe1a7f9b 8ee7eb4a 7c0f9e16 2bce3357 6b315ece  
cbb64068 37bf51f5

### D.1.2.4 Curve P-384

$p =$  3940200619639447921227904010014361380507973927046544666794  
8293404245721771496870329047266088258938001861606973112319

$n =$  3940200619639447921227904010014361380507973927046544666794  
6905279627659399113263569398956308152294913554433653942643

$SEED =$  a335926a a319a27a 1d00896a 6773a482 7acdac73

$c =$  79d1e655 f868f02f ff48dcde e14151dd b80643c1 406d0ca1  
0dfe6fc5 2009540a 495e8042 ea5f744f 6e184667 cc722483

$b =$  b3312fa7 e23ee7e4 988e056b e3f82d19 181d9c6e fe814112  
0314088f 5013875a c656398d 8a2ed19d 2a85c8ed d3ec2aef

$G_x =$  aa87ca22 be8b0537 8eb1c71e f320ad74 6e1d3b62 8ba79b98  
59f741e0 82542a38 5502f25d bf55296c 3a545e38 72760ab7

$G_y =$  3617de4a 96262c6f 5d9e98bf 9292dc29 f8f41dbd 289a147c  
e9da3113 b5f0b8c0 0a60b1ce 1d7e819d 7a431d7c 90ea0e5f

#### D.1.2.5 Curve P-521

$p =$  686479766013060971498190079908139321726943530014330540939  
 446345918554318339765605212255964066145455497729631139148  
 0858037121987999716643812574028291115057151  
 $n =$  686479766013060971498190079908139321726943530014330540939  
 446345918554318339765539424505774633321719753296399637136  
 3321113864768612440380340372808892707005449  
 $SEED =$  d09e8800 291cb853 96cc6717 393284aa a0da64ba  
 $c =$  0b4 8bfa5f42 0a349495 39d2bdfc 264eeeeb 077688e4  
 4fbf0ad8 f6d0edb3 7bd6b533 28100051 8e19f1b9 ffbe0fe9  
 ed8a3c22 00b8f875 e523868c 70cle5bf 55bad637  
 $b =$  051 953eb961 8e1c9a1f 929a21a0 b68540ee a2da725b  
 99b315f3 b8b48991 8ef109e1 56193951 ec7e937b 1652c0bd  
 3bb1bf07 3573df88 3d2c34f1 ef451fd4 6b503f00  
 $G_x =$  c6 858e06b7 0404e9cd 9e3ecb66 2395b442 9c648139  
 053fb521 f828af60 6b4d3dba a14b5e77 efe75928 fe1dc127  
 a2ffa8de 3348b3c1 856a429b f97e7e31 c2e5bd66  
 $G_y =$  118 39296a78 9a3bc004 5c8a5fb4 2c7d1bd9 98f54449  
 579b4468 17afbd17 273e662c 97ee7299 5ef42640 c550b901  
 3fad0761 353c7086 a272c240 88be9476 9fd16650

#### D.1.3 Curves over Binary Fields

For each field degree  $m$ , a pseudo-random curve is given, along with a Koblitz curve. The pseudo-random curve has the form

$$E: y^2 + xy = x^3 + x^2 + b,$$

and the Koblitz curve has the form

$$E_a: y^2 + xy = x^3 + ax^2 + 1,$$

where  $a = 0$  or  $1$ .

For each pseudorandom curve, the cofactor is  $h = 2$ . The cofactor of each Koblitz curve is  $h = 2$  if  $a = 1$ , and  $h = 4$  if  $a = 0$ .

The coefficients of the pseudo-random curves, and the coordinates of the base points of both kinds of curves, are given in terms of both the polynomial and normal basis representations discussed in Appendix D.1.1.3.

For each  $m$ , the following parameters are given:

*Field Representation:*

- The normal basis type  $T$
- The field polynomial (a trinomial or pentanomial)

*Koblitz Curve:*

- The coefficient  $a$
- The base point order  $n$
- The base point  $x$  coordinate  $G_x$
- The base point  $y$  coordinate  $G_y$

*Pseudo-random curve:*

- The base point order  $n$

*Pseudo-random curve (Polynomial Basis representation):*

- The coefficient  $b$
- The base point  $x$  coordinate  $G_x$
- The base point  $y$  coordinate  $G_y$

*Pseudo-random curve (Normal Basis representation):*

- The 160-bit input seed  $SEED$  to the SHA-1 based algorithm (i.e., the domain parameter seed)
- The coefficient  $b$  (i.e., the output of the SHA-1 based algorithm)
- The base point  $x$  coordinate  $G_x$
- The base point  $y$  coordinate  $G_y$

Integers (such as  $T$ ,  $m$ , and  $n$ ) are given in decimal form; bit strings and field elements are given in hexadecimal.

#### **D.1.3.1 Degree 163 Binary Field**

$$T = 4$$

$$p(t) = t^{163} + t^7 + t^6 + t^3 + 1$$

#### D.1.3.1.1 Curve K-163

$$a = 1$$

$$n = 5846006549323611672814741753598448348329118574063$$

*Polynomial Basis:*

$$G_x = 2 \text{ fe13c053 7bbc11ac aa07d793 de4e6d5e 5c94eee8}$$

$$G_y = 2 \text{ 89070fb0 5d38ff58 321f2e80 0536d538 ccdaa3d9}$$

*Normal Basis:*

$$G_x = 0 \text{ 5679b353 caa46825 fea2d371 3ba450da 0c2a4541}$$

$$G_y = 2 \text{ 35b7c671 00506899 06bac3d9 dec76a83 5591edb2}$$

#### D.1.3.1.2 Curve B-163

$$n = 5846006549323611672814742442876390689256843201587$$

*Polynomial Basis:*

$$b = 2 \text{ 0a601907 b8c953ca 1481eb10 512f7874 4a3205fd}$$

$$G_x = 3 \text{ f0eba162 86a2d57e a0991168 d4994637 e8343e36}$$

$$G_y = 0 \text{ d51fbc6c 71a0094f a2cdd545 b11c5c0c 797324f1}$$

*Normal Basis:*

$$SEED = 85e25bfe \text{ 5c86226c db12016f 7553f9d0 e693a268}$$

$$b = 6 \text{ 645f3cac f1638e13 9c6cd13e f61734fb c9e3d9fb}$$

$$G_x = 0 \text{ 311103c1 7167564a ce77ccb0 9c681f88 6ba54ee8}$$

$$G_y = 3 \text{ 33ac13c6 447f2e67 613bf700 9daf98c8 7bb50c7f}$$

#### D.1.3.2 Degree 233 Binary Field

$$T = 2$$

$$p(t) = t^{233} + t^{74} + 1$$



#### D.1.3.2.1 Curve K-233

$$a = 0$$

$$n = 345087317339528189371737793113851276057094098886225212 \backslash \\ 6328087024741343$$

*Polynomial Basis:*

$$G_x = 172 \ 32ba853a \ 7e731af1 \ 29f22ff4 \ 149563a4 \ 19c26bf5 \\ 0a4c9d6e \ efad6126$$

$$G_y = 1db \ 537dece8 \ 19b7f70f \ 555a67c4 \ 27a8cd9b \ f18aeb9b \\ 56e0c110 \ 56fae6a3$$

*Normal Basis:*

$$G_x = 0fd \ e76d9dcd \ 26e643ac \ 26f1aa90 \ 1aa12978 \ 4b71fc07 \\ 22b2d056 \ 14d650b3$$

$$G_y = 064 \ 3e317633 \ 155c9e04 \ 47ba8020 \ a3c43177 \ 450ee036 \\ d6335014 \ 34cac978$$

#### D.1.3.2.2 Curve B-233

$$n = 690174634679056378743475586227702555583981273734501355 \backslash \\ 5379383634485463$$

*Polynomial Basis:*

$$b = 066 \ 647ede6c \ 332c7f8c \ 0923bb58 \ 213b333b \ 20e9ce42 \\ 81fe115f \ 7d8f90ad$$

$$G_x = 0fa \ c9dfcbac \ 8313bb21 \ 39f1bb75 \ 5fef65bc \ 391f8b36 \\ f8f8eb73 \ 71fd558b$$

$$G_y = 100 \ 6a08a419 \ 03350678 \ e58528be \ bf8a0bef \ f867a7ca \\ 36716f7e \ 01f81052$$

*Normal Basis:*

$SEED =$  74d59ff0 7f6b413d 0ea14b34 4b20a2db 049b50c3  
 $b =$  1a0 03e0962d 4f9a8e40 7c904a95 38163adb 82521260  
 0c7752ad 52233279  
 $G_x =$  18b 863524b3 cdfefb94 f2784e0b 116faac5 4404bc91  
 62a363ba b84a14c5  
 $G_y =$  049 25df77bd 8b8ff1a5 ff519417 822bfedf 2bbd7526  
 44292c98 c7af6e02

### D.1.3.3 Degree 283 Binary Field

$T = 6$   
 $p(t) = t^{283} + t^{12} + t^7 + t^5 + 1$

#### D.1.3.3.1 Curve K-283

$a = 0$   
 $n =$  3885337784451458141838923813647037813284811733793061324  
 295874997529815829704422603873

*Polynomial Basis:*

$G_x =$  503213f 78ca4488 3f1a3b81 62f188e5 53cd265f 23c1567a  
 16876913 b0c2ac24 58492836  
 $G_y =$  1ccda38 0f1c9e31 8d90f95d 07e5426f e87e45c0 e8184698  
 e4596236 4e341161 77dd2259

*Normal Basis:*

$G_x =$  3ab9593 f8db09fc 188f1d7c 4ac9fcc3 e57fcd3b db15024b  
 212c7022 9de5fcd9 2eb0ea60  
 $G_y =$  2118c47 55e7345c d8f603ef 93b98b10 6fe8854f feb9a3b3  
 04634cc8 3a0e759f 0c2686b1

#### D.1.3.3.2 Curve B-283

$n =$  7770675568902916283677847627294075626569625924376904889  
109196526770044277787378692871

*Polynomial Basis:*

$b =$  27b680a c8b8596d a5a4af8a 19a0303f ca97fd76 45309fa2  
a581485a f6263e31 3b79a2f5

$G_x =$  5f93925 8db7dd90 e1934f8c 70b0dfec 2eed25b8 557eac9c  
80e2e198 f8cdbecd 86b12053

$G_y =$  3676854 fe24141c b98fe6d4 b20d02b4 516ff702 350eddb0  
826779c8 13f0df45 be8112f4

*Normal Basis:*

$SEED =$  77e2b073 70eb0f83 2a6dd5b6 2dfc88cd 06bb84be

$b =$  157261b 894739fb 5a13503f 55f0b3f1 0c560116 66331022  
01138cc1 80c0206b dafbc951

$G_x =$  749468e 464ee468 634b21f7 f61cb700 701817e6 bc36a236  
4cb8906e 940948ea a463c35d

$G_y =$  62968bd 3b489ac5 c9b859da 68475c31 5bafcdc4 ccd0dc90  
5b70f624 46f49c05 2f49c08c

#### D.1.3.4 Degree 409 Binary Field

$T =$  4

$p(t) = t^{409} + t^{87} + 1$

##### D.1.3.4.1 Curve K-409

$a =$  0

$n =$  33052798439512429947595765401638551991420234148214060964\  
232439502288071128924919105067325845777745801409636659061  
7731358671

*Polynomial Basis:*

$G_x =$  060f05f 658f49c1 ad3ab189 0f718421 0efd0987 e307c84c  
27accfb8 f9f67cc2 c460189e b5aaaa62 ee222eb1 b35540cf  
e9023746

$G_y =$  1e36905 0b7c4e42 acba1dac bf04299c 3460782f 918ea427  
e6325165 e9ea10e3 da5f6c42 e9c55215 aa9ca27a 5863ec48  
d8e0286b

*Normal Basis:*

$G_x =$  1b559c7 cba2422e 3affel33 43e808b5 5e012d72 6ca0b7e6  
a63aeafb cle3a98e 10ca0fcf 98350c3b 7f89a975 4a8eldc0  
713cec4a

$G_y =$  16d8c42 052f07e7 713e7490 eff318ba 1abd6fef 8a5433c8  
94b24f5c 817aeb79 852496fb ee803a47 bc8a2038 78ebf1c4  
99afd7d6

#### **D.1.3.4.2 Curve B-409**

$n =$  6610559687902485989519153080327710398284046829642812192  
84648798304157774827374805208143723762179110965979867288  
366567526771

*Polynomial Basis:*

$b =$  021a5c2 c8ee9feb 5c4b9a75 3b7b476b 7fd6422e flf3dd67  
4761fa99 d6ac27c8 a9a197b2 72822f6c d57a55aa 4f50ae31  
7b13545f

$G_x =$  15d4860 d088ddb3 496b0c60 64756260 441cde4a fl771d4d  
b01ffe5b 34e59703 dc255a86 8a118051 5603aeab 60794e54  
bb7996a7

$G_y =$  061b1cf ab6be5f3 2bbfa783 24ed106a 7636b9c5 a7bd198d  
0158aa4f 5488d08f 38514f1f df4b4f40 d2181b36 81c364ba  
0273c706

*Normal Basis:*

$SEED =$  4099b5a4 57f9d69f 79213d09 4c4bcd4d 4262210b  
 $b =$  124d065 1c3d3772 f7f5a1fe 6e715559 e2129bdf a04d52f7  
b6ac7c53 2cf0ed06 f610072d 88ad2fdc c50c6fde 72843670  
f8b3742a  
 $G_x =$  0ceacbc 9f475767 d8e69f3b 5dfab398 13685262 bcacf22b  
84c7b6dd 981899e7 318c96f0 761f77c6 02c016ce d7c548de  
830d708f  
 $G_y =$  199d64b a8f089c6 db0e0b61 e80bb959 34afd0ca f2e8be76  
d1c5e9af fc7476df 49142691 ad303902 88aa09bc c59c1573  
aa3c009a

#### **D.1.3.5 Degree 571 Binary Field**

$T =$  10

$p(t) = t^{571} + t^{10} + t^5 + t^2 + 1$

##### **D.1.3.5.1 Curve K-571**

$a =$  0

$n =$  1932268761508629172347675945465993672149463664853217499  
32861762572575957114478021226813397852270671183470671280  
08253514612736749740666173119296824216170925035557336852  
76673

*Polynomial Basis:*

$G_x =$  26eb7a8 59923fbc 82189631 f8103fe4 ac9ca297 0012d5d4  
60248048 01841ca4 43709584 93b205e6 47da304d b4ceb08c

bbd1ba39 494776fb 988b4717 4dca88c7 e2945283 a01c8972  
 $G_y =$  349dc80 7f4fbf37 4f4aeade 3bca9531 4dd58cec 9f307a54  
 ffc61efc 006d8a2c 9d4979c0 ac44aea7 4fbеbbbb9 f772aedc  
 b620b01a 7ba7af1b 320430c8 591984f6 01cd4c14 3ef1c7a3

*Normal Basis:*

$G_x =$  04bb2db a418d0db 107adae0 03427e5d 7cc139ac b465e593  
 4f0bea2a b2f3622b c29b3d5b 9aa7a1fd fd5d8be6 6057c100  
 8e71e484 bcd98f22 bf847642 37673674 29ef2ec5 bc3ebcf7  
 $G_y =$  44cbb57 de20788d 2c952d7b 56cf39bd 3e89b189 84bd124e  
 751ceff4 369dd8da c6a59e6e 745df44d 8220ce22 aa2c852c  
 fcbbef49 ebaa98bd 2483e331 80e04286 feaa2530 50caff60

#### **D.1.3.5.2 Curve B-571**

$n =$  3864537523017258344695351890931987344298927329706434998  
 65723525145151914228956042453614399938941577308313388112  
 19269444862468724628168130702345282883033324113931911052  
 85703

*Polynomial Basis:*

$b =$  2f40e7e 2221f295 de297117 b7f3d62f 5c6a97ff cb8ceff1  
 cd6ba8ce 4a9a18ad 84ffabbd 8efa5933 2be7ad67 56a66e29  
 4afdl85a 78ff12aa 520e4de7 39baca0c 7ffeff7f 2955727a  
 $G_x =$  303001d 34b85629 6c16c0d4 0d3cd775 0a93d1d2 955fa80a  
 a5f40fc8 db7b2abd bde53950 f4c0d293 cdd711a3 5b67fb14  
 99ae6003 8614f139 4abfa3b4 c850d927 e1e7769c 8eec2d19  
 $G_y =$  37bf273 42da639b 6dccfffe b73d69d7 8c6c27a6 009cbbca  
 1980f853 3921e8a6 84423e43 bab08a57 6291af8f 461bb2a8  
 b3531d2f 0485c19b 16e2f151 6e23dd3c 1a4827af 1b8ac15b

*Normal Basis:*

```

SEED = 2aa058f7 3a0e33ab 486b0f61 0410c53a 7f132310

b =      3762d0d 47116006 179da356 88eeacccf 591a5cde a7500011
      8d9608c5 9132d434 26101a1d fb377411 5f586623 f75f0000
      1ce61198 3c1275fa 31f5bc9f 4be1a0f4 67f01ca8 85c74777

Gx =      0735e03 5def5925 cc33173e b2a8ce77 67522b46 6d278b65
      0a291612 7dfea9d2 d361089f 0a7a0247 a184e1c7 0d417866
      e0fe0feb 0ff8f2f3 f9176418 f97d117e 624e2015 df1662a8

Gy =      04a3642 0572616c df7e606f ccadaecf c3b76dab 0eb1248d
      d03fbdfc 9cd3242c 4726be57 9855e812 de7ec5c5 00b4576a
      24628048 b6a72d88 0062eed0 dd34b109 6d3acbb6 b01a4a97

```

## D.2 Implementation of Modular Arithmetic

The prime moduli in the above examples are of a special type (called *generalized Mersenne numbers*) for which modular multiplication can be carried out more efficiently than in general. This section provides the rules for implementing this faster arithmetic for each of the prime moduli appearing in the examples.

The usual way to multiply two integers (mod  $m$ ) is to take the integer product and reduce it (mod  $m$ ). One therefore has the following problem: given an integer  $A$  less than  $m^2$ , compute

$$B = A \bmod m.$$

In general, one must obtain  $B$  as the remainder of an integer division. If  $m$  is a generalized Mersenne number, however, then  $B$  can be expressed as a sum or difference (mod  $m$ ) of a small number of terms. To compute this expression, the integer sum or difference can be evaluated and the result reduced modulo  $m$ . The latter reduction can be accomplished by adding or subtracting a few copies of  $m$ .

The prime modulus  $p$  for each of the five example curves is a generalized Mersenne number.

### D.2.1 Curve P-192

The modulus for this curve is  $p = 2^{192} - 2^{64} - 1$ . Every integer  $A$  less than  $p^2$  can be written as

$$A = A_5 \cdot 2^{320} + A_4 \cdot 2^{256} + A_3 \cdot 2^{192} + A_2 \cdot 2^{128} + A_1 \cdot 2^{64} + A_0,$$

where each  $A_i$  is a 64-bit integer. As a concatenation of 64-bit words, this can be denoted by

$$A = (A_5 \parallel A_4 \parallel A_3 \parallel A_2 \parallel A_1 \parallel A_0).$$

The expression for  $B$  is

$$B = T + S_1 + S_2 + S_3 \bmod p,$$

where the 192-bit terms are given by

$$T = (A_2 \parallel A_1 \parallel A_0)$$

$$S_1 = (A_3 \parallel A_3)$$

$$S_2 = (A_4 \parallel A_4 \parallel 0)$$

$$S_3 = (A_5 \parallel A_5 \parallel A_5).$$

### D.2.2 Curve P-224

The modulus for this curve is  $p = 2^{224} - 2^{96} + 1$ . Every integer  $A$  less than  $p^2$  can be written as:

$$A = A_{13} \cdot 2^{416} + A_{12} \cdot 2^{384} + A_{11} \cdot 2^{352} + A_{10} \cdot 2^{320} + A_9 \cdot 2^{288} + A_8 \cdot 2^{256} + A_7 \cdot 2^{224} + A_6 \cdot 2^{192} + A_5 \cdot 2^{160} + A_4 \cdot 2^{128} + A_3 \cdot 2^{96} + A_2 \cdot 2^{64} + A_1 \cdot 2^{32} + A_0,$$

where each  $A_i$  is a 32-bit integer. As a concatenation of 32-bit words, this can be denoted by:

$$A = (A_{13} \parallel A_{12} \parallel \dots \parallel A_0).$$

The expression for  $B$  is:

$$B = T + S_1 + S_2 - D_1 - D_2 \bmod p,$$

where the 224-bit terms are given by:

$$T = (A_6 \parallel A_5 \parallel A_4 \parallel A_3 \parallel A_2 \parallel A_1 \parallel A_0)$$

$$S_1 = (A_{10} \parallel A_9 \parallel A_8 \parallel A_7 \parallel 0 \parallel 0 \parallel 0)$$

$$S_2 = (0 \parallel A_{13} \parallel A_{12} \parallel A_{11} \parallel 0 \parallel 0 \parallel 0)$$

$$D_1 = (A_{13} \parallel A_{12} \parallel A_{11} \parallel A_{10} \parallel A_9 \parallel A_8 \parallel A_7)$$

$$D_2 = (0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{13} \parallel A_{12} \parallel A_{11}).$$

### D.2.3 Curve P-256

The modulus for this curve is  $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$ . Every integer  $A$  less than  $p^2$  can be written as:

$$A = A_{15} \cdot 2^{480} + A_{14} \cdot 2^{448} + A_{13} \cdot 2^{416} + A_{12} \cdot 2^{384} + A_{11} \cdot 2^{352} + A_{10} \cdot 2^{320} + A_9 \cdot 2^{288} + A_8 \cdot 2^{256} + A_7 \cdot 2^{224} + A_6 \cdot 2^{192} + A_5 \cdot 2^{160} + A_4 \cdot 2^{128} + A_3 \cdot 2^{96} + A_2 \cdot 2^{64} + A_1 \cdot 2^{32} + A_0,$$

where each  $A_i$  is a 32-bit integer. As a concatenation of 32-bit words, this can be denoted by

$$A = (A_{15} \parallel A_{14} \parallel \dots \parallel A_0).$$

The expression for  $B$  is:



$$B = T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4 \bmod p,$$

where the 256-bit terms are given by:

$$T = (A_7 \parallel A_6 \parallel A_5 \square \parallel A_4 \parallel A_3 \parallel A_2 \parallel A_1 \parallel A_0)$$

$$S_1 = (A_{15} \parallel A_{14} \square \parallel A_{13} \parallel A_{12} \parallel A_{11} \parallel 0 \parallel 0 \parallel 0)$$

$$S_2 = (0 \parallel A_{15} \parallel A_{14} \square \parallel A_{13} \parallel A_{12} \parallel 0 \parallel 0 \parallel 0)$$

$$S_3 = (A_{15} \parallel A_{14} \square \parallel 0 \parallel 0 \parallel 0 \parallel A_{10} \parallel A_9 \parallel A_8)$$

$$S_4 = (A_8 \parallel A_{13} \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{11} \parallel A_{10} \parallel A_9)$$

$$D_1 = (A_{10} \parallel A_8 \parallel 0 \parallel 0 \parallel 0 \parallel A_{13} \parallel A_{12} \parallel A_{11})$$

$$D_2 = (A_{11} \parallel A_9 \parallel 0 \parallel 0 \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{12} \square)$$

$$D_3 = (A_{12} \square \parallel 0 \parallel A_{10} \parallel A_9 \parallel A_8 \parallel A_{15} \parallel A_{14} \parallel A_{13})$$

$$D_4 = (A_{13} \parallel 0 \parallel A_{11} \parallel A_{10} \parallel A_9 \parallel 0 \parallel A_{15} \parallel A_{14})$$

#### D.2.4 Curve P-384

The modulus for this curve is  $p = 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1$ . Every integer  $A$  less than  $p^2$  can be written as:

$$\begin{aligned} A = & A_{23} \cdot 2^{736} + A_{22} \cdot 2^{704} + A_{21} \cdot 2^{672} + A_{20} \cdot 2^{640} + A_{19} \cdot 2^{608} + A_{18} \cdot 2^{576} + A_{17} \cdot 2^{544} + A_{16} \cdot 2^{512} + \\ & A_{15} \cdot 2^{480} + A_{14} \cdot 2^{448} + A_{13} \cdot 2^{416} + A_{12} \cdot 2^{384} + A_{11} \cdot 2^{352} + A_{10} \cdot 2^{320} + A_9 \cdot 2^{288} + A_8 \cdot 2^{256} + \\ & A_7 \cdot 2^{224} + A_6 \cdot 2^{192} + A_5 \cdot 2^{160} + A_4 \cdot 2^{128} + A_3 \cdot 2^{96} + A_2 \cdot 2^{64} + A_1 \cdot 2^{32} + A_0, \end{aligned}$$

where each  $A_i$  is a 32-bit integer. As a concatenation of 32-bit words, this can be denoted by

$$A = (A_{23} \parallel A_{22} \parallel \dots \parallel A_0).$$

The expression for  $B$  is:

$$B = T + 2S_1 + S_2 + S_3 + S_4 + S_5 + S_6 - D_1 - D_2 - D_3 \bmod p,$$

where the 384-bit terms are given by:

$$T = (A_{11} \parallel A_{10} \parallel A_9 \parallel A_8 \parallel A_7 \parallel A_6 \parallel A_5 \parallel A_4 \parallel A_3 \parallel A_2 \parallel A_1 \parallel A_0)$$

$$S_1 = (0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{23} \parallel A_{22} \parallel A_{21} \parallel 0 \parallel 0 \parallel 0 \parallel 0)$$

$$S_2 = (A_{23} \parallel A_{22} \parallel A_{21} \parallel A_{20} \parallel A_{19} \parallel A_{18} \parallel A_{17} \parallel A_{16} \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{12})$$

$$S_3 = (A_{20} \parallel A_{19} \parallel A_{18} \parallel A_{17} \parallel A_{16} \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{12} \parallel A_{23} \parallel A_{22} \parallel A_{21})$$

$$S_4 = (A_{19} \parallel A_{18} \parallel A_{17} \parallel A_{16} \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{12} \parallel A_{20} \parallel 0 \parallel A_{23} \parallel 0)$$

$$S_5 = (0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{23} \parallel A_{22} \parallel A_{21} \parallel A_{20} \parallel 0 \parallel 0 \parallel 0 \parallel 0)$$

$$\begin{aligned}
 S_6 &= (0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{23} \parallel A_{22} \parallel A_{21} \parallel 0 \parallel 0 \parallel A_{20}) \\
 D_1 &= (A_{22} \parallel A_{21} \parallel A_{20} \parallel A_{19} \parallel A_{18} \parallel A_{17} \parallel A_{16} \parallel A_{15} \parallel A_{14} \parallel A_{13} \parallel A_{12} \parallel A_{23}) \\
 D_2 &= (0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{23} \parallel A_{22} \parallel A_{21} \parallel A_{20} \parallel 0) \\
 D_3 &= (0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel A_{23} \parallel A_{23} \parallel 0 \parallel 0 \parallel 0).
 \end{aligned}$$

### D.2.5 Curve P-521

The modulus for this curve is  $p = 2^{521} - 1$ . Every integer  $A$  less than  $p^2$  can be written

$$A = A_1 \cdot 2^{521} + A_0,$$

where each  $A_i$  is a 521-bit integer. As a concatenation of 521-bit words, this can be denoted by

$$A = (A_1 \parallel A_0).$$

The expression for  $B$  is:

$$B = (A_0 + A_1) \bmod p.$$

### D.3 Normal Bases

The elements of  $GF(2^m)$  are expressed in terms of the type  $T$  normal basis<sup>5</sup>  $B$  for  $GF(2^m)$ , for some  $T$ . Each element has a unique representation as a bit string:

$$(a_0 \ a_1 \ \dots \ a_{m-1}).$$

The arithmetic operations are performed as follows.

Addition: addition of two elements is implemented by bit-wise addition modulo 2. Thus, for example,

$$(1100111) + (1010010) = (0110101).$$

Squaring: if

$$\alpha = (a_0 \ a_1 \ \dots \ a_{m-1})$$

then

$$\alpha^2 = (a_{m-1} \ a_0 \ a_1 \ \dots \ a_{m-2}).$$

Multiplication: to perform multiplication, a function  $F(\underline{u}, \underline{v})$  is constructed on inputs

---

<sup>5</sup> It is assumed in this section that  $m$  is odd and  $T$  is even, since this is the only case considered in this Standard.

$$\underline{u} = (u_0 \ u_1 \ \dots \ u_{m-1}) \quad \text{and} \quad \underline{v} = (v_0 \ v_1 \ \dots \ v_{m-1})$$

as follows.

1. Set  $p \leftarrow Tm + 1$ .
2. Let  $u$  be an integer having order  $T$  modulo  $p$ .
3. Compute the sequence  $F(1), F(2), \dots, F(p-1)$  as follows:
  - 3.1 Set  $w \leftarrow 1$ .
  - 3.2 For  $j$  from 0 to  $T-1$  do
    - 3.2.1 Set  $n \leftarrow w$ .
    - 3.2.2 For  $i = 0$  to  $m-1$  do
      - 3.2.2.1 Set  $F(n) \leftarrow i$ .
      - 3.2.2.2 Set  $n \leftarrow 2n \bmod p$ .
    - 3.2.3 Set  $w \leftarrow uw \bmod p$ .
4. Output the formula:

$$F(u, v) := \sum_{k=1}^{p-2} u_{F(k+1)} v_{F(p-k)}.$$

This computation need only be performed once per basis.

Given the function  $F$  for  $B$ , the product

$$(c_0 \ c_1 \ \dots \ c_{m-1}) = (a_0 \ a_1 \ \dots \ a_{m-1}) * (b_0 \ b_1 \ \dots \ b_{m-1})$$

is computed as follows:

1. Set  $(u_0 \ u_1 \ \dots \ u_{m-1}) \leftarrow (a_0 \ a_1 \ \dots \ a_{m-1})$ .
2. Set  $(v_0 \ v_1 \ \dots \ v_{m-1}) \leftarrow (b_0 \ b_1 \ \dots \ b_{m-1})$ .
3. For  $k = 0$  to  $m - 1$  do
  - 3.1 Compute
$$c_k = F(\underline{u}, \underline{v}).$$
  - 3.2 Set  $u \leftarrow \mathbf{LeftShift}(u)$  and  $v \leftarrow \mathbf{LeftShift}(v)$ , where **LeftShift** denotes the circular left shift operation.
4. Output  $c = (c_0 \ c_1 \ \dots \ c_{m-1})$ .

Example: For the type 4 normal basis for  $GF(2^7)$ ,  $p = 29$  and  $u = 12$  or  $17$ . Thus, the values of  $F$  are given by:

$$\begin{aligned} F(1) &= 0 & F(8) &= 3 & F(15) &= 6 & F(22) &= 5 \\ F(2) &= 1 & F(9) &= 3 & F(16) &= 4 & F(23) &= 6 \\ F(3) &= 5 & F(10) &= 2 & F(17) &= 0 & F(24) &= 1 \\ F(4) &= 2 & F(11) &= 4 & F(18) &= 4 & F(25) &= 2 \\ F(5) &= 1 & F(12) &= 0 & F(19) &= 2 & F(26) &= 5 \\ F(6) &= 6 & F(13) &= 4 & F(20) &= 3 & F(27) &= 1 \\ F(7) &= 5 & F(14) &= 6 & F(21) &= 3 & F(28) &= 0 \end{aligned}$$

Therefore,

$$\begin{aligned} F(\underline{u}, \underline{v}) &= u_0 v_1 + u_1 (v_0 + v_2 + v_5 + v_6) + u_2 (v_1 + v_3 + v_4 + v_5) + u_3 (v_2 + v_5) + \\ &\quad u_4 (v_2 + v_6) + u_5 (v_1 + v_2 + v_3 + v_6) + u_6 (v_1 + v_4 + v_5 + v_6). \end{aligned}$$

Thus, if

$$a = (1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1) \text{ and } b = (1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1),$$

then

$$\begin{aligned} c_0 &= F((1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1), (1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1)) = 1, \\ c_1 &= F((0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1), (1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1)) = 0, \\ &\quad \vdots \\ c_6 &= F((1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1), (1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0)) = 1, \end{aligned}$$

so that  $c = ab = (1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1)$ .

#### D.4 Scalar Multiplication on Koblitz Curves

This section describes a particularly efficient method of computing the scalar multiple  $nP$  on the Koblitz curve  $E_a$  over  $GF(2^m)$ .

The operation  $\tau$  is defined by:

$$\tau(x, y) = (x^2, y^2).$$

When the normal basis representation is used, then the operation  $\tau$  is implemented by performing right circular shifts on the bit strings representing  $x$  and  $y$ .

Given  $m$  and  $a$ , define the following parameters:

- $C$  is some integer greater than 5.

- $\mu = (-1)^{1-a}$ .
- For  $i = 0$  and  $i = 1$ , define the sequence  $s_i(m)$  by:

$$s_i(0) = 0, \quad s_i(1) = 1 - i,$$

$$s_i(m) = \mu \bullet s_i(m-1) - 2 s_i(m-2) + (-1)^i$$

- Define the sequence  $V(m)$

$$V(0) = 2, \quad V(1) = \mu$$

$$V(m) = \mu \bullet v(m-1) - 2V(m-2).$$

For the example curves, the quantities  $s_i(m)$  and  $V(m)$  are as follows.

Curve K-163:

$$s_0(163) = 2579386439110731650419537$$

$$s_1(163) = -755360064476226375461594$$

$$V(163) = -4845466632539410776804317$$

Curve K-233:

$$s_0(233) = -27859711741434429761757834964435883$$

$$s_1(233) = -44192136247082304936052160908934886$$

$$V(233) = -137381546011108235394987299651366779$$

Curve K-283:

$$s_0(283) = -665981532109049041108795536001591469280025$$

$$s_1(283) = 1155860054909136775192281072591609913945968$$

$$V(283) = 7777244870872830999287791970962823977569917$$

Curve K-409:

$$s_0(409) = -18307510456002382137810317198756461378590542487556869338419259$$

$$s_1(409) = -8893048526138304097196653241844212679626566100996606444816790$$

$$V(409) = 10457288737315625927447685387048320737638796957687575791173829$$

Curve K-571:

$$s_0(571) = -3737319446876463692429385892476115567147293964596131024123406420 \backslash$$

$$235241916729983261305$$

$$s_1(571) = -3191857706446416099583814595948959674131968912148564658610565117 \backslash$$

$$58982848515832612248752$$

V(571)= -1483809269816914138996191402970514903645425741804939362329123395\  
34208516828973111459843

The following algorithm computes the scalar multiple  $nP$  on the Koblitz curve  $E_a$  over  $GF(2^m)$ . The average number of elliptic additions and subtractions is at most  $\sim 1 + (m/3)$ , and is at most  $\sim m/3$  with probability at least  $1 - 2^{5-C}$ .

1. For  $i = 0$  to 1 do
  - 1.1  $n' \leftarrow \lfloor n / 2^{a-C + (m-9)/2} \rfloor$ .
  - 1.2  $g' \leftarrow s_i(m) \cdot n'$ .
  - 1.3  $h' \leftarrow \lfloor g' / 2^m \rfloor$ .
  - 1.4  $j' \leftarrow V(m) \cdot h'$ .
  - 1.5  $l' \leftarrow \text{Round}((g' + j') / 2^{(m+5)/2})$ .
  - 1.6  $\lambda_i \leftarrow l' / 2^C$ .
  - 1.7  $f_i \leftarrow \text{Round}(\lambda_i)$ .
  - 1.8  $\eta_i \leftarrow \lambda_i - f_i$ .
  - 1.9  $h_i \leftarrow 0$ .
2.  $\eta \leftarrow 2 \eta_0 + \mu \eta_1$ .
3. If  $(\eta \geq 1)$ ,
 

then

  - if  $(\eta_0 - 3 \mu \eta_1 < -1)$ 

then set  $h_1 \leftarrow \mu$

else set  $h_0 \leftarrow 1$ .

else

  - if  $(\eta_0 + 4 \mu \eta_1 \geq 2)$ 

then set  $h_1 \leftarrow \mu$ .
4. If  $(\eta < -1)$ 

then

  - if  $(\eta_0 - 3 \mu \eta_1 \geq 1)$ 

then set  $h_1 \leftarrow -\mu$

else set  $h_0 \leftarrow -1$ .

else  
     if  $(\eta_0 + 4\mu\eta_1 < -2)$   
         then set  $h_1 \leftarrow -\mu$ .

5.  $q_0 \leftarrow f_0 + h_0$ .
6.  $q_1 \leftarrow f_1 + h_1$ .
7.  $r_0 \leftarrow n - (s_0 + \mu s_1) q_0 - 2s_1 q_1$ .
8.  $r_1 \leftarrow s_1 q_0 - s_0 q_1$ .
9. Set  $Q \leftarrow O$ .
10.  $P_0 \leftarrow P$ .
11. While  $((r_0 \neq 0) \text{ or } (r_1 \neq 0))$ 
  - 11.1 If  $(r_0 \text{ odd})$ , then
    - 11.1.1 set  $u \leftarrow 2 - (r_0 - 2r_1 \bmod 4)$ .
    - 11.1.2 set  $r_0 \leftarrow r_0 - u$ .
    - 11.1.3 if  $(u = 1)$ , then set  $Q \leftarrow Q + P_0$ .
    - 11.1.4 if  $(u = -1)$ , then set  $Q \leftarrow Q - P_0$ .
  - 11.2 Set  $P_0 \leftarrow \tau P_0$ .
  - 11.3 Set  $(r_0, r_1) \leftarrow (r_1 + \mu r_0 / 2, -r_0 / 2)$ .
- Endwhile
12. Output  $Q$ .

## D.5 Generation of Pseudo-Random Curves (Prime Case)

Let  $l$  be the bit length of  $p$ , and define

$$v = \lfloor (l - 1) / 160 \rfloor$$

$$w = l - 160v - 1.$$

1. Choose an arbitrary 160-bit string  $s$  as the domain parameter seed.
2. Compute  $h = \text{SHA-1}(s)$ .
3. Let  $h_0$  be the bit string obtained by taking the  $w$  rightmost bits of  $h$ .
4. Let  $z$  be the integer whose binary expansion is given by the 160-bit string  $s$ .
5. For  $i$  from 1 to  $v$  do:

5.1 Define the 160-bit string  $s_i$  to be binary expansion of the integer

$$(z + i) \bmod (2^{160}).$$

5.2 Compute  $h_i = \text{SHA-1}(s_i)$ .

6. Let  $h$  be the bit string obtained by the concatenation of  $h_0, h_1, \dots, h_v$  as follows:

$$h = h_0 \parallel h_1 \parallel \dots \parallel h_v.$$

7. Let  $c$  be the integer whose binary expansion is given by the bit string  $h$ .

8. If  $((c = 0 \text{ or } 4c + 27 \equiv 0 \pmod{p}))$ , then go to Step 1.

9. Choose integers  $a, b \in GF(p)$  such that

$$c b^2 \equiv a^3 \pmod{p}.$$

(The simplest choice is  $a = c$  and  $b = c$ . However, one may want to choose differently for performance reasons.)

10. Check that the elliptic curve  $E$  over  $GF(p)$  given by  $y^2 = x^3 + ax + b$  has suitable order. If not, go to Step 1.

## D.6 Verification of Curve Pseudo-Randomness (Prime Case)

Given the 160-bit domain parameter seed value  $s$ , verify that the coefficient  $b$  was obtained from  $s$  via the cryptographic hash function SHA-1 as follows.

Let  $l$  be the bit length of  $p$ , and define

$$v = \lfloor (l - 1) / 160 \rfloor$$

$$w = l - 160v - 1.$$

1. Compute  $h = \text{SHA-1}(s)$ .

2. Let  $h_0$  be the bit string obtained by taking the  $w$  rightmost bits of  $h$ .

3. Let  $z$  be the integer whose binary expansion is given by the 160-bit string  $s$ .

4. For  $i = 1$  to  $v$  do

4.1 Define the 160-bit string  $s_i$  to be binary expansion of the integer

$$(z + i) \bmod (2^{160}).$$

4.2 Compute  $h_i = \text{SHA-1}(s_i)$ .

5. Let  $h$  be the bit string obtained by the concatenation of  $h_0, h_1, \dots, h_v$  as follows:

$$h = h_0 \parallel h_1 \parallel \dots \parallel h_v.$$



6. Let  $c$  be the integer whose binary expansion is given by the bit string  $h$ .
7. Verify that  $b^2 c \equiv -27 \pmod{p}$ .

#### D.7 Generation of Pseudo-Random Curves (Binary Case)

Let:

$$v = \lfloor (m - 1) / B \rfloor$$

$$w = m - Bv.$$

1. Choose an arbitrary 160-bit string  $s$  as the domain parameter seed.
2. Compute  $h = \text{SHA-1}(s)$ .
3. Let  $h_0$  be the bit string obtained by taking the  $w$  rightmost bits of  $h$ .
4. Let  $z$  be the integer whose binary expansion is given by the 160-bit string  $s$ .
5. For  $i$  from 1 to  $v$  do:
  - 5.1 Define the 160-bit string  $s_i$  to be binary expansion of the integer  $(z + i) \bmod (2^{160})$ .
  - 5.2 Compute  $h_i = \text{SHA-1}(s_i)$ .
6. Let  $h$  be the bit string obtained by the concatenation of  $h_0, h_1, \dots, h_v$  as follows:

$$h = h_0 \parallel h_1 \parallel \dots \parallel h_v.$$

7. Let  $b$  be the element of  $GF(2^m)$  which binary expansion is given by the bit string  $h$ .
8. Choose an element  $a$  of  $GF(2^m)$ .
9. Check that the elliptic curve  $E$  over  $GF(2^m)$  given by  $y^2 + xy = x^3 + ax^2 + b$  has suitable order. If not, go to Step 1.

#### D.8 Verification of Curve Pseudo-Randomness (Binary Case)

Given the 160-bit domain parameter seed value  $s$ , verify that the coefficient  $b$  was obtained from  $s$  via the cryptographic hash function SHA-1 as follows.

Define

$$v = \lfloor (m - 1) / 160 \rfloor$$

$$w = m - 160v$$

1. Compute  $h = \text{SHA-1}(s)$ .
2. Let  $h_0$  be the bit string obtained by taking the  $w$  rightmost bits of  $h$ .

3. Let  $z$  be the integer whose binary expansion is given by the 160-bit string  $s$ .
4. For  $i = 1$  to  $v$  do
  - 4.1 Define the 160-bit string  $s_i$  to be binary expansion of the integer  $(z + i) \bmod (2^{160})$ .
  - 4.2 Compute  $h_i = \text{SHA-1}(s_i)$ .
5. Let  $h$  be the bit string obtained by the concatenation of  $h_0, h_1, \dots, h_v$  as follows:

$$h = h_0 \parallel h_1 \parallel \dots \parallel h_v.$$

6. Let  $c$  be the element of  $GF(2^m)$  which is represented by the bit string  $h$ .
7. Verify that  $c = b$ .

## D.9 Polynomial Basis to Normal Basis Conversion

Suppose that  $\alpha$  is an element of the field  $GF(2^m)$ . Let  $p$  be the bit string representing  $\alpha$  with respect to a given polynomial basis. It is desired to compute  $n$ , the bit string representing  $\alpha$  with respect to a given normal basis. This is done via the matrix computation

$$p \Gamma = n,$$

where  $\Gamma$  is an  $m$ -by- $m$  matrix with entries in  $GF(2)$ . The matrix  $\Gamma$ , which depends only on the bases, can be computed easily given its second-to-last row. The second-to-last row for each conversion is given the below.

Degree 163:

3 e173bfaf 3a86434d 883a2918 a489ddbd 69fe84e1

Degree 233:

0be 19b89595 28bbc490 038f4bc4 da8bdfc1 ca36bb05 853fd0ed  
0ae200ce

Degree 283:

3347f17 521fdabc 62ec1551 acf156fb 0bceb855 f174d4c1 7807511c  
9f745382 add53bc3

Degree 409:

0eb00f2 ea95fd6c 64024e7f 0b68b81f 5ff8a467 acc2b4c3 b9372843  
6265c7ff a06d896c ae3a7e31 e295ec30 3eb9f769 de78bef5

Degree 571:

7940ffa ef996513 4d59dcbf e5bf239b e4fe4b41 05959c5d 4d942ffd  
46ea35f3 e3cdb0e1 04a2aa01 cef30a3a 49478011 196bfb43 c55091b6  
1174d7c0 8d0cdd61 3bf6748a bad972a4

Given the second-to-last row  $r$  of  $\Gamma$ , the rest of the matrix is computed as follows. Let  $\beta$  be the element of  $GF(2^m)$  whose representation with respect to the normal basis is  $r$ . Then the rows of  $\Gamma$ , from top to bottom, are the bit strings representing the elements

$$\beta^{m-1}, \beta^{m-2}, \dots, \beta^2, \beta, 1$$

with respect to the normal basis. (Note that the element 1 is represented by the all-1 bit string.)

Alternatively, the matrix is the inverse of the matrix described in Appendix D.10.

More details of these computations can be found in Annex A.7 of the IEEE Standard 1363-2000 standard.

## D.10 Normal Basis to Polynomial Basis Conversion

Suppose that  $\alpha$  is an element of the field  $GF(2^m)$ . Let  $n$  be the bit string representing  $\alpha$  with respect to a given normal basis. It is desired to compute  $p$ , the bit string representing  $\alpha$  with respect to a given polynomial basis. This is done via the matrix computation

$$n \Gamma = p,$$

where  $\Gamma$  is an  $m$ -by- $m$  matrix with entries in  $GF(2)$ . The matrix  $\Gamma$ , which depends only on the bases, can be computed easily given its top row. The top row for each conversion is given below.

### Degree 163:

7 15169c10 9c612e39 0d347c74 8342bcd3 b02a0bef

### Degree 233:

149 9e398ac5 d79e3685 59b35ca4 9bb7305d a6c0390b cf9e2300  
253203c9

### Degree 283:

31e0ed7 91c3282d c5624a72 0818049d 053e8c7a b8663792 bc1d792e  
ba9867fc 7b317a99

### Degree 409:

0dfa06b e206aa97 b7a41fff b9b0c55f 8f048062 fbe8381b 4248adf9  
2912ccc8 e3f91a24 elcfb395 0532b988 971c2304 2e85708d

### Degree 571:

452186b bf5840a0 bcf8c9f0 2a54efa0 4e813b43 c3d41496 06c4d27b  
487bf107 393c8907 f79d9778 beb35ee8 7467d328 8274caeb da6ce05a  
eb4ca5cf 3c3044bd 4372232f 2c1a27c4

Given the top row  $r$  of  $\Gamma$ , the rest of the matrix is computed as follows. Let  $\beta$  be the element of  $GF(2^m)$  whose representation with respect to the polynomial basis is  $r$ . Then the rows of  $\Gamma$ , from top to bottom, are the bit strings representing the elements

$$\beta, \beta^2, \beta^{2^2}, \dots, \beta^{2^{m-1}}$$

with respect to the polynomial basis.

Alternatively, the matrix is the inverse of the matrix described in Appendix D.9.

More details of these computations can be found in Annex A.7 of the IEEE Std 1363-2000 standard.

## Appendix E: A Proof that $v = r$ in the DSA (Informative)

The purpose of this appendix is to show that if  $M' = M$ ,  $r' = r$  and  $s' = s$  in the signature verification, then  $v = r'$ . Let **Hash** be an approved hash function. The following result is needed.

**Lemma:** Let  $p$  and  $q$  be primes such that  $q$  divides  $(p - 1)$ , let  $h$  be a positive integer less than  $p$ , and let  $g = (h^{(p-1)/q} \bmod p)$ . Then  $(g^q \bmod p) = 1$ , and if  $(m \bmod q) = (n \bmod q)$ , then  $(g^m \bmod p) = (g^n \bmod p)$ .

Proof:

$$\begin{aligned} g^q \bmod p &= (h^{(p-1)/q} \bmod p)^q \bmod p \\ &= h^{(p-1)} \bmod p \\ &= 1 \end{aligned}$$

by Fermat's Little Theorem. Now let  $(m \bmod q) = (n \bmod q)$ , i.e.,  $m = (n + kq)$  for some integer  $k$ . Then

$$\begin{aligned} g^m \bmod p &= g^{n+kq} \bmod p \\ &= (g^n g^{kq}) \bmod p \\ &= ((g^n \bmod p) (g^q \bmod p)^k) \bmod p \\ &= g^n \bmod p, \\ &\text{since } (g^q \bmod p) = 1. \end{aligned}$$

Proof of the main result:

**Theorem:** If  $M' = M$ ,  $r' = r$ , and  $s' = s$  in the signature verification, then  $v = r'$ .

Proof:

$$\begin{aligned} w &= (s')^{-1} \bmod q = s^{-1} \bmod q \\ u1 &= ((\text{Hash}(M'))w) \bmod q = ((\text{Hash}(M))w) \bmod q \\ u2 &= ((r')w) \bmod q = (rw) \bmod q. \end{aligned}$$

Now  $y = (g^x \bmod p)$ , so that by the lemma,

$$\begin{aligned} v &= ((g^{u1} y^{u2}) \bmod p) \bmod q \\ &= ((g^{\text{Hash}(M)w} y^{rw}) \bmod p) \bmod q \\ &= ((g^{\text{Hash}(M)w} g^{xrw}) \bmod p) \bmod q \\ &= ((g^{(\text{Hash}(M) + xr)w}) \bmod p) \bmod q. \end{aligned}$$

Also:

$$s = (k^{-1} (\mathbf{Hash}(M) + xr)) \bmod q.$$

Hence:

$$\begin{aligned} w &= (k (\mathbf{Hash}(M) + xr)^{-1}) \bmod q \\ (\mathbf{Hash}(M) + xr)w \bmod q &= k \bmod q. \end{aligned}$$

Thus, by the lemma:

$$v = (g^k \bmod p) \bmod q = r$$

## Appendix F: Calculating the Required Number of Rounds of Testing Using the Miller-Rabin Probabilistic Primality Test (Informative)

### F.1 The Required Number of Rounds of the Miller-Rabin Primality Tests

The ideas of paper [1] were applied to estimate  $p_{k,t}$ , the probability that an odd  $k$ -bit integer that passes  $t$  rounds of Miller-Rabin (M-R) testing is actually composite. The probability  $p_{k,t}$  is understood as the ratio of the number of odd composite numbers of a binary length  $k$  that can be expected to pass  $t$  rounds of M-R testing (with randomly generated bases) to the sum of that value and the number of odd prime integers of binary length  $k$ . This is equivalent to assuming that candidates selected for testing will be chosen uniformly at random from the entire set of odd  $k$ -bit integers. Following Pomerance, et al.,  $p_{k,t}$  can be (over) estimated by the ratio of the expected number of odd composite numbers of binary length  $k$  that will pass  $t$  rounds of M-R testing (with randomly generated bases) to the total number of odd primes of binary length  $k$ . From the perspective of a party charged with the responsibility of generating a  $k$ -bit prime, the objective is to determine a value of  $t$  such that  $p_{k,t}$  is no greater than an acceptably small target value  $p_{target}$ .

Using [1], it is possible to compute an upper bound for  $p_{k,t}$  as a function of  $k$  and  $t$ . From this, an upper bound can be computed for  $t$  as a function of  $k$  and  $p_{target}$ , the maximum allowed probability of accidentally generating a composite number. The following is an algorithm for computing  $t$ :

1. For  $t = 1, 2 \dots \lceil -\log_2(p_{target})/2 \rceil$ 
  - 1.1 For  $M = 3, 4 \dots \lfloor 2\sqrt{k-1} - 1 \rfloor$  (1)
    - 1.1.1 Compute  $p_{k,t}$  as in (2).
    - 1.1.2 If  $p_{k,t} \leq p_{target}$ 
      - 1.1.2.1 Accept  $t$ .
      - 1.1.2.2 Stop.

In (1),  $k$  is the bit length of the candidate primes and (2) is as follows:

$$p_{k,t} = 2.00743 \cdot \ln(2) \cdot k \cdot 2^{-k} \left[ 2^{k-2-Mt} + \frac{8(\pi^2 - 6)}{3} 2^{k-2} \sum_{m=3}^M 2^{m-(m-1)t} \sum_{j=2}^m \frac{1}{2^{\left(j + \frac{(k-1)}{j}\right)}} \right]. \quad (2)$$

Using this expression for  $t$ , the following methodologies are used for testing the DSA and RSA candidate primes.

## F.2 Generating DSA Primes

For DSA, the maximum possible care must be taken when generating the primes  $p$  and  $q$  that are used for the domain parameters. The same primes  $p$  and  $q$  are used by many parties. This means that any weakness that these numbers may possess would affect multiple users. It also means that the primes are not generated very often; typically, an entire system uses the same set of domain parameters for an extended period of time. Therefore, in this case, some additional care is called for.

With this in mind, it may be too optimistic to simply subject candidate primes to  $t$  rounds of M-R testing, where the minimal acceptable value for  $t$  is determined according to (1) and (2) in Appendix F.1. This might be the case, for example, if there is a reason to doubt that the assumptions made in [1] have been satisfied during the process of selecting candidates for primality testing. One may gain more confidence in the process by performing some additional (different) primality test(s) on the candidates that survive the M-R testing. As another option, one could, of course, perform additional rounds of M-R testing. These considerations lead to the following alternatives: either (A) use the number of rounds of M-R testing determined according to (1) and (2) in Appendix F.1, and follow that with a single Lucas test (as recommended in ANS X9.31), or (B) use a (much) more conservative approach when determining  $t$  (e.g., as described below) and subject candidate primes to additional rounds of M-R testing.

One approach for strategy (B) would be to adopt the viewpoint of the majority of system users, who have no part in generating the (supposed) prime, but who must rely upon its primality for their security. Such parties may be concerned that the candidates for M-R testing have been selected in a fashion that deviates significantly from the uniform distribution – which was assumed when determining  $t$  according to (1) and (2) in Appendix F.1. In cases where the selection process could be unusually biased in some way, it is important to minimize the probability that a composite number will survive testing. It can be shown that for any  $k$ -bit odd composite number (regardless of how it was selected), the probability that it will pass  $t$  rounds of M-R testing with randomly chosen bases is less than  $4^{-t}$  (although this is not a particularly tight bound). Selecting  $t$  such that  $4^{-t} \leq p_{\text{target}}$  is equivalent to choosing  $t \geq -\log_2(p_{\text{target}})/2$ . To ensure that a composite number has a probability no greater than  $p_{\text{target}}$  of surviving the M-R tests, the number of rounds can be set at  $t = \lceil -\log_2(p_{\text{target}})/2 \rceil$ . Even if the method of selecting candidates were so biased that it offered nothing but composite numbers for testing, it is reasonable to expect that it would take at least  $1/p_{\text{target}}$  attempts (which is greater than  $4^t$ ) before a composite number would slip through the  $t$ -round M-R testing process.

**WARNING:** As the discussion above illustrates, care must be taken when using the phrase “error probability” in connection with the recommended number of rounds of M-R testing. The probability that a composite number survives  $t$  rounds of Miller-Rabin testing is not the same as



$p_{k,t}$ , which is the probability that a number surviving  $t$  rounds of Miller-Rabin testing is composite. Ordinarily, the latter probability is the one that should be of most interest to a party responsible for generating primes, while the former may be more important to a party responsible for validating the primality of a number generated by someone else. However, for sufficiently large  $k$  (e.g.,  $k \geq 51$ ), it can be shown that  $p_{k,t} \leq 4^{-t}$  under the same assumptions concerning the selection of candidates as those made to obtain formula (2) in Appendix F.1 (see [1].) In such cases,  $t = \lceil -\log_2(p_{\text{target}})/2 \rceil$  rounds of Miller-Rabin testing can be used both in generating and validating primes, with  $p_{\text{target}}$  serving as an upper bound on both the probability that the generation process yields a composite number and the probability that a composite number would survive an attempt to validate its primality.

Table C.1 in Appendix C.3 identifies the minimum values for  $t$  when generating the primes  $p$  and  $q$  for DSA using either strategy (A) or (B) above. To obtain the  $t$  values shown in the column titled “M-R Tests Only”, the conservative strategy (B) was followed; those  $t$  values are sufficient to validate the primality of  $p$  and  $q$ . The  $t$  values shown in the column titled “M-R Tests when followed by One Lucas Test” result from following strategy (A) using computations (1) and (2) in Appendix F.1.

### F.3 Generating Primes for RSA Signatures

When generating primes for the RSA signature algorithm, it is still very important to reduce the probability of errors in the M-R testing procedure. However, since the (probable) primes are used to generate a user’s key pair, if a composite number survives the testing process, the consequences of the error may be less dramatic than in the case of generating DSA domain parameters; only one user’s transactions are affected, rather than a domain of users. Furthermore, if the  $p$  or  $q$  value generated for some user is composite, the problem will not be undiscovered for long, since it is almost certain that signatures generated by that user will not be verifiable.

Therefore, when generating the RSA primes  $p$  and  $q$ , it is sufficient to use the number of rounds derived from (1) and (2) in Appendix F.1 as the minimum number of M-R tests to be performed. However, if the definition of  $p_{k,t}$  is not considered to be sufficiently conservative when testing  $p$  and  $q$ , it is recommended that the  $t$  rounds of Miller-Rabin tests be followed by a single Lucas test.

The lengths for  $p$  and  $q$  that are recommended for use in RSA signature algorithms are 512, 1024 and 1536 bits; recall that  $n = pq$ , so the corresponding lengths for  $n$  are 1024, 2048 and 3072 bits, respectively. As currently specified in SP 800-57, Part 1, these lengths correspond to security strengths of 80, 112 and 128 bits, respectively. Hence, it makes sense to match the number of rounds of Miller-Rabin testing to the target error probability values of  $2^{-80}$ ,  $2^{-112}$ , and  $2^{-128}$ . A probability of  $2^{-100}$  is included for all prime lengths, since this probability has often been used in the past and may be acceptable for many applications.

When generating the RSA primes  $p$  and  $q$  with conditions, it is sufficient to use the value  $t$  derived from (1) and (2) as the minimum number of M-R tests to be performed when generating

the auxiliary primes  $p_1, p_2, q_1$  and  $q_2$ . It is not necessary to use an additional Lucas test on these numbers. In the extremely unlikely event that one of the numbers  $p_1, p_2, q_1$  or  $q_2$  is composite, there is still a high probability that the corresponding RSA prime ( $p$  or  $q$ ) will satisfy the requisite conditions.

The sizes of  $p_1, p_2, q_1$ , and  $q_2$  were chosen to ensure that, for an adversary with significant but not overwhelming resources, Lenstra's elliptic curve factoring method [2] (against which there is no protection beyond choosing large  $p$  and  $q$ ) is a more effective factoring algorithm than either the Pollard P-1 method [2], the Williams P+1 method [3] or various cycling methods [2]. For an adversary with overwhelming resources, the best all-purpose factoring algorithm is assumed to be the General Number Field Sieve [2].

Tables C.2 and C.3 in Appendix C.3 specify the minimum number of rounds of M-R testing when generating primes to be used in the construction of RSA signature key pairs.

## Appendix G: References

- [1] I. Damgard, P. Landrock, and C. Pomerance, C. “Average Case Error Estimates for the Strong Probable Prime Test,” *Mathematics of Computation*, v. 61, No, 203, pp. 177-194, 1993.
- [2] A.J Menezes, P.C. Oorschot, and S.A. Vanstone. *Handbook of Applied Cryptography*. CRC Press, 1996.
- [3] H.C. Williams. “A  $p+1$  Method of factoring”. *Math. Comp.* 39, 225-234, 1982.
- [4] D.E. Knuth, *The Art of Computer Programming*, Vol. 2, 3<sup>rd</sup> Ed., Addison-Wesley, 1998, Algorithm P, page 395.
- [5] R. Baillie and S.S. Wagstaff Jr., *Mathematics of Computation*, V. 35 (1980), pages 1391 – 1417.

Microsoft Press  
**Computer  
Dictionary**

Third Edition

**Microsoft** Press

PUBLISHED BY

Microsoft Press  
A Division of Microsoft Corporation  
One Microsoft Way  
Redmond, Washington 98052-6399

Copyright © 1997 by Microsoft Corporation

All rights reserved. No part of the contents of this book may be reproduced or transmitted in any form or by any means without the written permission of the publisher.

Library of Congress Cataloging-in-Publication Data pending.

ISBN 1-57231-743-4

Printed and bound in the United States of America.

1 2 3 4 5 6 7 8 9 QMQM 2 1 0 9 8 7

Distributed to the book trade in Canada by Macmillan of Canada, a division of Cañada Publishing Corporation.

A CIP catalogue record for this book is available from the British Library.

Microsoft Press books are available through booksellers and distributors worldwide. For further information about international editions, contact your local Microsoft Corporation office. Or contact Microsoft Press International directly at fax (425) 936-7329. Visit our Web site at [mspress.microsoft.com](http://mspress.microsoft.com).

Macintosh, Power Macintosh, QuickTime, and TrueType fonts are registered trademarks of Apple Computer, Inc. Intel is a registered trademark of Intel Corporation. DirectInput, DirectX, Microsoft, Microsoft Press, MS-DOS, Visual Basic, Visual C++, Win32, Win32s, Windows, Windows NT, and XENIX are registered trademarks and ActiveMovie, ActiveX, and Visual J++ are trademarks of Microsoft Corporation. Java is a trademark of Sun Microsystems, Inc. Other product and company names mentioned herein may be the trademarks of their respective owners.

**Acquisitions Editor:** Kim Fryer

**Project Editors:** Maureen Williams Zimmerman, Anne Taussig

**Technical Editors:** Dail Magee Jr., Gary Nelson, Jean Ross, Jim Fuchs, John Conrow,  
Kurt Meyer, Robert Lyon, Roslyn Lutsch

**compare**

a circuit that compares two input voltages and indicates which is higher.

**compare** \kəm-pār\ *vb.* To check two items, such as words, files, or numeric values, so as to determine whether they are the same or different. In a program, the outcome of a compare operation often determines which of two or more actions is taken next.

**compatibility** \kəm-pat'ə-bil'ə-tē\ *n.* **1.** The degree to which a computer, an attached device, a data file, or a program can work with or understand the same commands, formats, or language as another. True compatibility means that any operational differences are invisible to people and programs alike. **2.** The extent to which two machines can work in harmony. Compatibility (or the lack thereof) between two machines indicates whether, and to what degree, the computers can communicate, share data, or run the same programs. For example, an Apple Macintosh and an IBM PC are generally incompatible because they cannot communicate freely or share data without the aid of hardware and/or software that functions as an intermediary or a converter. **3.** The extent to which a piece of hardware conforms to an accepted standard (for example, IBM-compatible or Hayes-compatible). In this sense, compatibility means that the hardware ideally operates in all respects like the standard on which it is based. **4.** In reference to software, harmony on a task-oriented level among computers and computer programs. Computers deemed software-compatible are those that can run programs originally designed for other makes or models. Software compatibility also refers to the extent to which programs can work together and share data. In another area, totally different programs, such as a word processor and a drawing program, are compatible with one another if each can incorporate images or files created using the other. All types of software compatibility become increasingly important as computer communications, networks, and program-to-program file transfers become near-essential aspects of microcomputer operation. *See also* downward compatibility, upward-compatible.

**compatibility box** \kəm-pat'ə-bil'ə-tē boks\ *n.* *See* DOS box (definition 1).

**compiler**

**compatibility mode** \kəm-pat'ə-bil'ə-tē mōd\ *n.* A mode in which hardware or software in one system supports operations of software from another system. The term often refers to the ability of advanced operating systems designed for Intel microprocessors (for example, OS/2 and Windows NT) to run MS-DOS software or to the ability of some UNIX workstations and of some Apple Macintosh systems to run MS-DOS software.

**compile** \kəm-pīl\ *vb.* To translate all the source code of a program from a high-level language into object code prior to execution of the program. Object code is executable machine code or a variation of machine code. More generally, *compiling* is sometimes used to describe translating any high-level symbolic description into a lower-level symbolic or machine-readable format. A program that performs this task is known as a *compiler*. *See also* compile time, compiler (definition 2), high-level language, machine code, source code. *Compare* interpret.

**compile-and-go** \kəm-pīl'ənd-gō\ *adj.* Of, pertaining to, or characteristic of a development environment that automatically runs a program after compiling it. *See also* compile, execute.

**compiled Basic** \kəm-pīld' bā'sik, B-A-S-I-C\ *n.* Any version of Basic that is translated into machine code prior to execution by a compiler. Basic has traditionally been an interpreted language (translated and executed statement by statement); because compiled Basic generally produces faster-executing programs, it is the technology of choice for professional Basic programmers. *See also* Basic, compiled language, interpreted language.

**compiled language** \kəm-pīld' lang'wəj\ *n.* A language that is translated into machine code prior to any execution, as opposed to an interpreted language, which is translated and executed statement by statement. *See also* compiler (definition 2). *Compare* interpreted language.

**compiler** \kəm-pī'lər\ *n.* **1.** Any program that transforms one set of symbols into another by following a set of syntactic and semantic rules. **2.** A program that translates all the source code of a program written in a high-level language into object code prior to execution of the program. *See also* assembler, compile, high-level language,

# **McGraw-Hill Dictionary of Scientific and Technical Terms**

## **Fifth Edition**

**Sybil P. Parker**  
Editor in Chief

**McGraw-Hill, Inc.**

Auckland	Bogotá	New York	San Francisco	Washington, D.C.	Mexico City	Milan
Montreal	New Delhi	Caracas	Lisbon	London	Sydney	Toronto
		San Juan	Singapore			

On the cover: Photomicrograph of crystals of vitamin B<sub>1</sub>.  
(Dennis Kunkel, University of Hawaii)

Included in this Dictionary are definitions which have been published previously in the following works: P. B. Jordain, *Condensed Computer Encyclopedia*, Copyright © 1969 by McGraw-Hill, Inc. All rights reserved. J. Markus, *Electronics and Nucleonics Dictionary*, 4th ed., Copyright © 1960, 1966, 1978 by McGraw-Hill, Inc. All rights reserved. J. Quick, *Artists' and Illustrators' Encyclopedia*, Copyright © 1969 by McGraw-Hill, Inc. All rights reserved. *Blakiston's Gould Medical Dictionary*, 3d ed., Copyright © 1956, 1972 by McGraw-Hill, Inc. All rights reserved. T. Baumeister and L. S. Marks, eds., *Standard Handbook for Mechanical Engineers*, 7th ed., Copyright © 1958, 1967 by McGraw-Hill, Inc. All rights reserved.

In addition, material has been drawn from the following references: R. E. Huschke, *Glossary of Meteorology*, American Meteorological Society, 1959; *U.S. Air Force Glossary of Standardized Terms*, AF Manual 11-1, vol. 1, 1972; *Communications-Electronics Terminology*, AF Manual 11-1, vol. 3, 1970; W. H. Allen, ed., *Dictionary of Technical Terms for Aerospace Use*, 1st ed., National Aeronautics and Space Administration, 1965; J. M. Gilliland, *Solar-Terrestrial Physics: A Glossary of Terms and Abbreviations*, Royal Aircraft Establishment Technical Report 67158, 1967; *Glossary of Air Traffic Control Terms*, Federal Aviation Agency; *A Glossary of Range Terminology*, White Sands Missile Range, New Mexico, National Bureau of Standards, AD 467-424; *A DOD Glossary of Mapping, Charting and Geodetic Terms*, 1st ed., Department of Defense, 1967; P. W. Thrush, comp. and ed., *A Dictionary of Mining, Mineral, and Related Terms*, Bureau of Mines, 1968; *Nuclear Terms: A Glossary*, 2d ed., Atomic Energy Commission; F. Casey, ed., *Compilation of Terms in Information Sciences Technology*, Federal Council for Science and Technology, 1970; *Glossary of Stinfo Terminology*, Office of Aerospace Research, U.S. Air Force, 1963; *Naval Dictionary of Electronic, Technical, and Imperative Terms*, Bureau of Naval Personnel, 1962; *ADP Glossary*, Department of the Navy, NAVSO P-3097.

#### **McGRAW-HILL DICTIONARY OF SCIENTIFIC AND TECHNICAL TERMS, Fifth Edition**

Copyright © 1994, 1989, 1984, 1978, 1976, 1974 by McGraw-Hill, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 0 DOW/DOW 9 9 8 7 6 5 4 3

ISBN 0-07-042333-4

#### **Library of Congress Cataloging-in-Publication Data**

McGraw-Hill dictionary of scientific and technical terms /

Sybil P. Parker, editor in chief.—5th ed.

p. cm.

ISBN 0-07-042333-4

1. Science—Dictionaries. 2. Technology—Dictionaries.

I. Parker, Sybil P.

Q123.M34 1993

503—dc20

93-34772

CIP

#### **INTERNATIONAL EDITION**

Copyright © 1994. Exclusive rights by McGraw-Hill, Inc. for manufacture and export. This book cannot be re-exported from the country to which it is consigned by McGraw-Hill. The International Edition is not available in North America.

When ordering this title, use ISBN 0-07-113584-7.



**peptone** [BIOCHEM] A water-soluble mixture of proteoses and amino acids derived from albumin, meat, or milk; used as a nutrient and to prepare nutrient media for bacteriology. { 'pep,tŏn }

**per-** [CHEM] Prefix meaning: 1. Complete, as in hydrogen peroxide. 2. Extreme, or the presence of the peroxy (—O—O—) group. 3. Exhaustive (complete) substitution, as in perchloroethylene. { pər or per }

**Peracarida** [INV ZOO] A superorder of the Eumalacostraca; these crustaceans have the first thoracic segment united with the head, the cephalothorax usually larger than the abdomen, and some thoracic segments free from the carapace. { ,pə-rə'kar-ə-də }

**peracetic acid** [ORG CHEM]  $\text{CH}_3\text{COOOH}$  A toxic, colorless liquid with strong aroma; boils at  $105^\circ\text{C}$ ; explodes at  $110^\circ\text{C}$ ; miscible with water, alcohol, glycerin, and ether; used as an oxidizer, bleach, catalyst, bactericide, fungicide, epoxy-resin precursor, and chemical intermediate. Also known as peroxyacetic acid. { ,pə-rə'sēd-ik 'as-əd }

**peracid** [CHEM] Acid containing the peroxy (—O—O—) group, such as peracetic acid or perchloric acid. { 'pər'as-əd }

**peralcohol** [ORG CHEM] Chemical compound containing the peroxy group (—O—O—), such as peracetic acid and perchromic acid. { pər'al-kə,həl }

**peralkaline** [PETR] Of igneous rock, having a molecular proportion of aluminum lower than that of sodium oxide and potassium oxide combined. { pər'al-kə,līn }

**peraluminous** [PETR] Of igneous rock, having a molecular proportion of aluminum oxide greater than that of sodium oxide and potassium oxide combined. { ,pə-rə'lū-mə-nəs }

**Peramelidae** [VERT ZOO] The bandicoots, a family of insectivorous mammals in the order Marsupialia. { ,pə-rə'mel-ə,dē }

**P-E ratio** See precipitation-evaporation ratio. { 'pē'ē ,rā-shō }

**perazine** [PHARM]  $\text{C}_{20}\text{H}_{25}\text{N}_3\text{S}$  A crystalline compound, melting at  $51-53^\circ\text{C}$ ; used in medicine as a tranquilizer. { 'pər-ə,zēn }

**perbenzoic acid** [ORG CHEM]  $\text{C}_6\text{H}_5\text{CO}_2\text{OH}$  A crystalline compound forming leaflets from benzene solution, melting at  $41-45^\circ\text{C}$ , freely soluble in organic solvents; used in analysis of unsaturated compounds and to change ethylinic compounds into oxides. { ,pər-ben'zō-ik 'as-əd }

**perbituminous** [GEOL] Referring to bituminous coal containing more than 5.8% hydrogen, analyzed on a dry, ash-free basis. { ,pər-bə'tū-mə-nəs }

**percale** [TEXT] A plain-weave, medium-weight fabric made of cotton or cotton and polyester. { pər'kāl }

**percaline** [TEXT] Fine cotton with a glossy or moiré surface. { ,pər-kə'lēn }

**perceived noise decibel** [ACOUS] A unit of perceived noise level. Abbreviated PNdB. { pər'sēvd 'nōiz 'des-ə,bel }

**perceived noise level** [ACOUS] In perceived noise decibels, the noise level numerically equal to the sound pressure level, in decibels, of a band of random noise of width one-third to one octave centered on a frequency of 1000 hertz which is judged by listeners to be equally noisy. { pər'sēvd 'nōiz ,lev-əl }

**percent** [MATH] A quantitative term whereby  $n$  percent of a number is  $n$  one-hundredths of the number. Symbolized %. { pər'sent }

**percentage** [MATH] The result obtained by taking a given percent of a given quantity. { pər'sen-tij }

**percentage depletion** [PETRO ENG] Oil- or gas-reservoir depletion allowance calculated on the basis of unit sales and initial depletable leasehold cost. { pər'sen-tij di'plē-shən }

**percentage depth dose** [NUCLEO] The percentage of the absorbed dose from ionizing radiation at a given depth within a body to the absorbed dose at a reference point, usually the position of the peak absorbed dose. { pər'sen-tij 'depth 'dōs }

**percentage differential relay** [ELECTR] Differential relay which functions when the difference between two quantities of the same nature exceeds a fixed percentage of the smaller quantity. Also known as biased relay; ratio-balance relâ; ratio-differential relay. { pər'sen-tij ,dif-ə'ren-chəl 'rē,lā }

**percentage distribution** [STAT] A frequency distribution in which the individual class frequencies are expressed as a percentage of the total frequency equated to 100. Also known as relative frequency distribution; relative frequency table. { pər'sen-tij ,dis-trə'byū-shən }

**percentage extraction** [MIN ENG] The proportion of a coal

seam or other ores which is removed from a mine. { pər'sen-tij ik'strak-shən }

**percentage log** [ENG] A sample log in which the percentage of each type of rock (except obvious cavings) present in each sample of cuttings is estimated and plotted. { pər'sen-tij ,lāg }

**percentage map** [PETRO ENG] Contoured map in which the percentage of one component (such as sand) is compared with the total unit (such as sand plus shale). { pər'sen-tij ,map }

**percentage modulation** See percent modulation. { pər'sen-tij ,maj-ə'lā-shən }

**percentage ripple** [ELECTR] Ratio of the effective value of the ripple voltage to the average value of the total voltage, expressed as a percentage. { pər'sen-tij 'rip-əl }

**percentage subsidence** [MIN ENG] The measured amount of subsidence expressed as a percentage of the thickness of coal extracted. { pər'sen-tij səb'sid-əns }

**percentage support** [MIN ENG] The percentage of the total wall area which will actually be covered by supports. { pər'sen-tij sə'pōrt }

**percent compaction** [ENG] The ratio, expressed as a percentage, of dry unit weight of a soil to maximum unit weight obtained in a laboratory compaction test. { pər'sent kəm'pak-shən }

**percent defective** [IND ENG] The ratio of defective pieces per lot or sample, expressed as a percentage. { pər'sent di'fek-div }

**percent distortion** [COMMUN] The ratio of the amplitude of a harmonic component to the fundamental component multiplied by 100. { pər'sent di'stōr-shən }

**percentile** [STAT] A value in the range of a set of data which separates the range into two groups so that a given percentage of the measures lies below this value. { pər'sen,tīl }

**percent make** [ELECTR] 1. In pulse testing, the length of time a circuit stands closed compared to the length of the test signal. 2. Percentage of time during a pulse period that telephone dial pulse springs are making contact. { pər'sent 'māk }

**percent modulation** [COMMUN] The modulation factor expressed as a percentage. Also known as percentage modulation. { pər'sent ,maj-ə'lā-shən }

**percept** [PSYCH] A mental image of something perceived. { 'pər,sept }

**perception** [PHYSIO] Recognition in response to sensory stimuli; the act or process by which the memory of certain qualities of an object is associated with other qualities impressing the senses, thereby making possible recognition of the object. { pər'sep-shən }

**perception** [COMPUT SCI] A pattern recognition machine, based on an analogy to the human nervous system, capable of learning by means of a feedback system which reinforces correct answers and discourages wrong ones. { pər'sep,trən }

**perceptual overload** [PHYSIO] Saturation of the nervous system by an input of excess sensory information, resulting in an absence of response. { pər'sep-chə-wəl 'ō-vər,lōd }

**perch** [MECH] Also known as pole; rod. 1. A unit of length, equal to 5.5 yards, or 16.5 feet, or 5.0292 meters. 2. A unit of area, equal to 30.25 square yards, or 272.25 square feet, or 25.29285264 square meters. [NAV] A staff placed on top of a buoy, rock, or shoal as a mark for navigators; a ball or cage is sometimes placed at the top of the perch, as an identifying mark. [VERT ZOO] 1. Any member of the family Percidae. 2. The common name for a number of unrelated species of fish belonging to the Centrarchidae, Anabantoidae, and Percopsiformes. { pərch }

**perched aquifer** [HYD] An aquifer that is separated from another water-bearing stratum by an impermeable layer. { 'pərcht 'ak-wə-fər }

**perched block** [GEOL] A large, detached rock fragment presumed to have been transported and deposited by a glacier, and perched in a conspicuous and precarious position on the side of a hill. Also known as balanced rock; perched boulder; perched rock. { 'pərcht 'blāk }

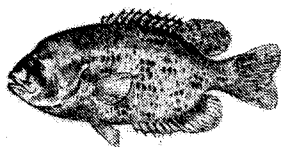
**perched boulder** See perched block. { 'pərcht 'bōl-dər }

**perched groundwater** See perched water. { 'pərcht 'graund,wōd-ər }

**perched lake** [HYD] A perennial lake whose surface level lies at a considerably higher elevation than those of other bodies of water, including aquifers, directly or closely associated with the lake. { 'pərcht 'lāk }

**perched rock** See perched block. { 'pərcht 'rāk }

PERCH



Rock bass (*Ambloplites rupestris*), a perch of the family Centrarchidae.

*The* **HUTCHINSON**  
DICTIONARY OF  
**COMPUTING**  
**MULTIMEDIA**  
AND THE  
**INTERNET**  
**THIRD EDITION**



Helicon

Text and original illustrations © Helicon Publishing Ltd 1997, 1998, 1999

First Published 1997

Second Edition 1998

Third Edition 1999

All rights reserved

Helicon Publishing Ltd

42 Hythe Bridge Street

Oxford OX1 2EP

e-mail: [admin@helicon.co.uk](mailto:admin@helicon.co.uk)

Web Site: <http://www.helicon.co.uk>

ISBN 1-85986-287-X

### **British Cataloguing in Publication Data**

A catalogue record for this book is available from the British Library

Terms which are known to be trademarks or service marks have been appropriately capitalized.

The use of a term in this book should not be regarded as affecting the validity or legal status of any proprietary rights which may be claimed in that trademark or service mark.

Printed and bound in Great Britain by  
The Bath Press Ltd, Bath, Somerset

## **Acknowledgements**

We would like to thank the following for screen shots of proprietary software: Adobe (Illustrator), MegaTech Software GmbH (MegaCAD), Microsoft Corp. (Excel, Internet Explorer, PowerPoint, Word), Netscape (Navigator); and the following WWW pages:  
AltaVista (<http://www.altavista.digital.com/>),  
Infoseek (<http://guide-p.infoseek.com/>),  
Snap (<http://www.snap.com/>),  
Lycos (<http://www.lycos.com/>),  
Pepsi (<http://www.pepsi.com/>),  
Jennicam (<http://www.jennicam.com/>),  
Yahoo (<http://www.yahoo.com/>).

**PAL** abbreviation for *Phase Alternation by Line*, video standard used in the UK, other parts of Europe, and China. It has a higher definition and different screen format from the US NTSC standard. Running a television program written for PAL on an NTSC system can result in the bottom of the screen image being cut off.

**palmtop computer** or *palm-sized PC*, palm-sized computer that usually does not have a keyboard but is operated by using a stylus on a touch-sensitive screen. Although several companies launched palmtops and personal digital assistants, the first successful model was Palm Computing's Palm Pilot. (Palm was taken over by modem manufacturer US Robotics, which in turn was taken over by networking company 3Com.) In 1998, Casio, Philips, and other companies launched Palm-like palmtops running Microsoft's Windows CE operating system.

**Pantone Matching System** trade name for a standard set of colours used in graphics and printing, precisely graded and numbered using a universal system. Colours shown on monitors and computer printers are not reliable enough for most designers, so print graphics programs usually include the facility to specify the desired Pantone colours.

**parallax** in virtual reality, the distance between the viewer's left and right eyes in the virtual world.

This difference is what creates the impression of depth. Right and left images of distant objects look the same; but left and right images of nearby objects look markedly different owing to the difference of perspective. Manipulating this variable in a virtual world helps make objects look large or small.

**parallel device** a device that communicates binary data by sending the bits that represent each character simultaneously along a set of separate data lines, unlike a serial device.

**parallel processing** emerging computer technology that allows more than one computation at the same time. Although in the 1980s this technology enabled only a small number of computer processor units to work in parallel, in theory thousands or millions of processors could be used at the same time.

Parallel processing, which involves breaking down computations into small parts and performing thousands of them simultaneously, rather than in a linear sequence, offers the prospect of a vast improvement in working speed for certain repetitive applications.

**parallel running** a method of implementing a new computer system in which the new system and the old system are run together for a short while. The old system is therefore available to take over from its replacement should any faults arise. An alternative method is pilot running.

**parameter** variable factor or characteristic. For example, length is one parameter of a rectangle; its height is another. In computing, it is frequently useful to describe a program or object with a set of variable parameters rather than fixed values.

## 210 PARITY

For example, if a programmer writes a routine for drawing a rectangle using general parameters for the length, height, line thickness, and so on, any rectangle can be drawn by this routine by giving different values to the parameters.

Similarly, in a word-processing application that stores parameters for font, page layout, type of justification, and so on, these can be changed by the user.

**parity** of a number, the state of being either even or odd. In computing, the term refers to the number of 1s in the binary codes used to represent data. A binary representation has **even parity** if it contains an even number of 1s and **odd parity** if it contains an odd number of 1s.

For example, the binary code 1000001, commonly used to represent the character 'A', has even parity because it contains two 1s, and the binary code 1000011, commonly used to represent the character 'C', has odd parity because it contains three 1s. A **parity bit** (0 or 1) is sometimes added to each binary representation to give all the same parity so that a validation check can be carried out each time data are transferred from one part of the computer to another. So, for example, the codes 1000001 and 1000011 could have parity bits added and become 01000001 and 11000011, both with even parity. If any bit in these codes should be altered in the course of processing the parity would change and the error would be quickly detected.

**parity**  
Examples of parity for  
different letters of the  
alphabet (represented by  
binary code).

Character	Binary code	Parity	Base-ten representation
A	1000001	even	65
B	1000010	even	66
C	1000011	odd	67
D	1000100	even	68

**parity check** a form of validation of data.

**parse** software facility for breaking down a data stream into individual pieces of information that can be acted upon. On the World Wide Web, for example, data entered by a user can be sent to a database program for storage and later analysis; the ability to do this depends on being able to feed the right bit of data into the right record field.

**PASCAL** French acronym for *program appliqué à la selection et la compilation automatique de la littérature*, a high-level computer-programming language. Designed by Niklaus Wirth in the 1960s as an aid to teaching programming, it is still widely used as such in universities, and as a good general-purpose programming language. Most professional programmers, however, now use C or C++. PASCAL was named after 17th-century French mathematician Blaise Pascal.

**Pascal, Blaise** (1623–1666) French mathematician and philosopher, inventor of the first mechanical calculator able to carry 'tens' automatically when adding

IEEE Std 100-1996

# **The IEEE Standard Dictionary of Electrical and Electronics Terms**

**Sixth Edition**

**Standards Coordinating Committee 10, Terms and Definitions  
Jane Radatz, Chair**

This standard is one of a number of information technology dictionaries being developed by standards organizations accredited by the American National Standards Institute. This dictionary was developed under the sponsorship of voluntary standards organizations, using a consensus-based process.

I:

SH94459  
1-55937-833-6  
8 April 1997



## signal contrast

**signal contrast (facsimile)** The ratio expressed in decibels between white signal and black signal. *See also:* facsimile signal. (COM) 168-1956w

**signal converter (test, measurement, and diagnostic equipment)** A device for changing a signal from one form or value to another form or value. (MIL) [2]

**signal current (diode-type camera tube)** The change in target current which occurs when the target is irradiated with photons, or electrons, compared to the case where no radiation is incident on the target. (ED) 503-1978w

**signal decay time (measuring the performance of tone address signaling systems)** The time interval between the end of the signal present condition and the beginning of the signal off condition at the end of the signal under consideration. (COM) 752-1986r

**signal decorrelation time** *See:* decorrelation time.

**signal delay** The transmission time of a signal through a network. The time is always finite, may be undesired, or may be purposely introduced. *See also:* delay line; oscillograph. (IM) [40]

**signal, difference** *See:* differential signal.

**signal distance (1) (computers)** The number of digit positions in which the corresponding digits of two binary words of the same length are different. *See also:* hamming distance. (COM/C) [20], 312-1977w

(2) (mathematics of computing) *See also:* hamming distance. (C) 1084-1986w

**signal distributing (telephone switching systems)** Delivering of signals from a common control to other circuits. (COM) 312-1977w

**signal duration (measuring the performance of tone address signaling systems)** The time interval during which a signal present condition exists continuously. (COM) 752-1986r

**signal electrode (camera tubes)** An electrode from which the signal output is obtained. *See also:* electrode. (BT) [34]

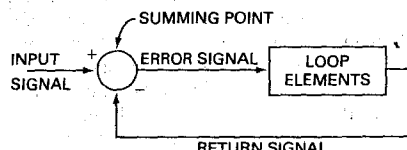
**signal element (1) (data transmission) (unit interval)** The part of a signal that occupies the shortest interval of signaling code. It is considered to be of unit duration in building up signal combinations. (PE) 599-1985w

(2) A signal element is the logical signal value during one half of a bit time and may take on the values of Logic-1 or Logic-0. (C/LM) 8802-5-1995

**signal, error (1) (automatic control device)** A signal whose magnitude and sign are used to correct the alignment between the controlling and the controlled elements. (EEC/PE) [119]

(2) (power supplies) The difference between the output voltage and a fixed reference voltage compared in ratio by the two resistors at the null junction of the comparison bridge. The error signal is amplified to drive the pass elements and correct the output. (AE/PE) [41], [78]

(3) (closed loop) (control system feedback) The signal resulting from subtracting a particular return signal from its corresponding input signal. (See the corresponding figure.) *See also:* control system, feedback.



block diagram of a closed loop

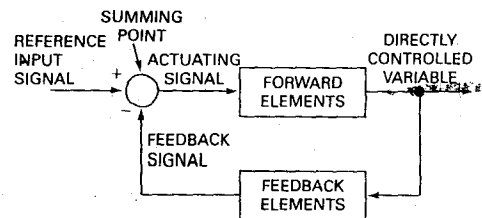
## signal, error

(PE) [3], 421-1972s

**signal, feedback (1) (general)** A function of the directly controlled variable in such form as to be used at the summing point. *See also:* control system, feedback. (IA) [60]

(2) (control system feedback) The return signal that results from the reference input signal. (See the corresponding figure.) *See also:* control system, feedback.

## signaling circuit



Simplified block diagram including essential elements of an automatic control system

## signal, feedback

(PE) [3], 421-1972s

**signal flow graph (network analysis)** A network of directed branches in which each dependent node signal is the algebraic sum of the incoming branch signals at that node. *Note:* Thus,

$$x_1 t_{1k} + x_2 t_{2k} + \dots + x_n t_{nk} = x_k$$

at each dependent node  $k$ , where  $t_{jk}$  is the branch transmittance of branch  $jk$ . (CAS) 155-1960r

**signal frequency shift (frequency-shift facsimile system)** The numerical difference between the frequencies corresponding to white signal and black signal at any point in the system. *See also:* facsimile signal. (COM) 168-1956w

**signal generator** A shielded source of voltage or power, the output level and frequency of which are calibrated, and usually variable over a range. *Note:* The output of known waveform is normally subject to one or more forms of calibrated modulation. (IM) [40]

**signal ground** For the purpose of this guide, shall be the grounding system to which signals are referenced. (PE) 1050-1996

**signal identifier (spectrum analyzer)** A means to identify the frequency of the input when spurious responses are possible. A front panel control used to identify the input frequency when spurious responses are present. (IM) 748-1979w

**signal indication** The information conveyed by the aspect of a signal. (EEC/PE) [119]

**signaling (1) (data transmission)** The production of an audible or visible signal at a station or switchboard by means of an alternating or pulsating current. In a telephone system, any of several methods used to alert subscribers or operators or to establish and control connections. (PE) 599-1985w

(2) (telephone switching systems) The transmission of address and other switching information between stations and central offices and between switching entities. (COM) 312-1977w

(3) The exchange of information specifically concerned with the establishment and control of connections, and the transfer of user-to-user and management information in a circuit-switched network. (C/LM) 802.9a-1995

(4) The exchange of information specifically concerned with the establishment and control of connections, and the transfer of user-to-user and management information in a telecommunication network, e.g., in a PPSN. (COM/C/LM) 8802-9-1996

**signaling, analog** A means of communicating between devices that uses continuously variable signals. (PE/SUB) 999-1992

**signaling and doorbell transformers (power and distribution transformers)** Step-down transformers (having a secondary of 30 V or less), generally used for the operation of signals, chimes, and doorbells. (PE) C57.12.80-1978r

**signaling, binary** A means of communicating between devices that uses two-state signals. Where multiple binary data bits are to be transferred, either multiple signaling paths ("parallel binary") or a time series of individual data bits ("serial binary") transmission methods are to be used. (PE/SUB) 999-1992

**signaling circuit** Any electric circuit that energizes signaling equipment. (NEC/NESC) [86]



## signaling light

990

## signal queueing

**signaling light (illuminating engineering)** A projector used for directing light signals toward a designated target zone.

(EEC/IE) [126]

**signal, input (control system feedback)** A signal applied to a system or element. See the figure attached to definition "3" of signal, error. *See also:* control system, feedback.

(PE) [3], 421-1972s

**signal integration** The summation of a succession of signals by writing them at the same location on the storage surface. *See also:* storage tube.

(ED) 158-1962w

**signal interphasing** A method of simultaneously overlapping multiple transmission signals to achieve higher transmission rates.

(C) 610.7-1995

**signal layer** The layer of the protocol stack at which signals are specified.

(BA/C) 1355-1995

**signal level (1) (696 interface devices) (signals and paths)** The magnitude of a signal when considered in relation to an arbitrary reference magnitude.

(C/IM/MM) 488.1-1987r, 696-1983w

(2) **(SBX bus)** The relative magnitude of a signal when compared to a reference.

(C/MM) 959-1988r

(3) **(STEBus)** The relative magnitude of a signal when considered in relation to an arbitrary reference. The unit of representation is the volt.

(C/MM) 1000-1987r, 796-1983r

(4) **(broadband local area networks)** The measured voltage or power of a signal usually stated in dBmV.

(C/LM) 802.7-1989

**signal line (1) (696 interface devices) (microcomputer system bus) (programmable instrumentation) (signals and paths)** One of a set of signal conductors in an interface system used to transfer messages among interconnected devices.

(C/IM/MM) 488.1-1987r, 696-1983w, 796-1983r, 959-1988r

(2) **(STEBus)** One of a set of signal conductors in an interface system used to transfer data among interconnected boards.

(C/MM) 1000-1987r

(3) **(NuBus)** A conductor on the backplane other than ground, or power.

(C/MM) 1196-1987

(4) An electrical or optical information-carrying facility, such as a differential pair of wires or an optical fiber, with associated transmitter and receiver, carrying binary true/false logic values.

(C/MM) 1596-1992

(5) An electrical or optical information-carrying facility, such as a differential pair of wires or an optical fiber, with associated driver and receiver, carrying binary true/false logic values.

(C/MM) 1596.3-1996

**signal lines** The passive transmission lines through which the signal passes from one to another of the elements of the signal transmission system. *See also:* signal.

(IE) [43]

**signal names (1)** Where a group of bus lines are represented by the same letters, the lines within the group are numbered, e.g., AD0\*, AD1\*, AD2\*, etc. In order to represent a group of lines or signals in more convenient form, the notation AD[31..0]\* is used. Also, the notation AD[\*] is used to refer to all of the lines within the group.

(BA/C) 896.2-1991

(2) Where a group of bus lines are represented by the same letters, the lines within the group are numbered; e.g., AD0\*, AD1\*, AD2\*, etc. In order to represent a group of lines or signals in more convenient form, notation such as AD[63..0]\* is used. Also, in these examples, the notation AD[\*] is used to refer to all of the lines within the group.

(BA/C) 10857-1994

(3) Where a group of bus lines are represented by the same letters, the lines within the group are numbered, e.g., AD0\*, AD1\*, AD2\*, etc. In order to represent a group of lines or signals in more convenient form, the notation AD[63..0]\* is used. Also, the notation AD[\*] is used to refer to all of the lines within the group.

(BA/C) 896.4-1993

**signal negation** The act of driving a signal to the false state.

(BA/C) 1496-1993

**signal off (measuring the performance of tone address signaling systems)** Any condition where all the constituent tones

of a tone signaling system are below a specified OFF level for each tone. In a single-tone signaling system, tone off and signal off are synonymous terms. *Note:* During a signal off condition, tones that are not used in the signaling system may be at a higher level.

(COM) 752-1986r

**signal operation (elevators)** Operation by means of single buttons or switches (or both) in the car, and up-or-down direction buttons (or both) at the landings, by which predetermined landing stops may be set up or registered for an elevator or for a group of elevators. The stops set up by the momentary actuation of the car buttons are made automatically in succession as the car reaches those landings, irrespective of its direction of travel or the sequence in which the buttons are actuated. The stops set up by the momentary actuation of the up-and-down buttons at the landing are made automatically by the first available car in the group approaching the landing in the corresponding direction, irrespective of the sequence in which the buttons are actuated. With this type of operation, the car can be started only by means of a starting switch or button in the car. *See also:* control.

(EEC/PE) [119]

**signal, output (control system feedback)** A signal delivered by a system or element. *See also:* control system, feedback.

(NESC/PE) [86], 421-1972s

**signal output current (camera tubes or phototubes)** The absolute value of the difference between output current and dark current. *See also:* phototube.

(BT/ED) [34], [45], 161-1971w

**signal parameter (programmable instrumentation)** That parameter of an electrical quantity whose values or sequence of values convey information.

(C/IM/MM) 1000-1987r, 488.1-1987r, 696-1983w, 796-1983r

**signal phase** The initial phase of an exception operation in which all agents are notified of an error condition. *See also:* exception operation.

(C/MM) 1296-1987s

**signal present (measuring the performance of tone address signaling systems)** Any condition where the presence of tone or tones is sufficient to be recognized as a valid digital or supervisory signal. In a single-tone signaling system, tone present and signal present are synonymous terms; in a two-tone signaling system the signal present state exists where two and only two tones each meet the signal present condition, and the two tones represent a valid combination.

(COM) 752-1986r

**signal processing antenna system** An antenna system having circuit elements associated with its radiating element(s) that perform functions such as multiplication, storage, correlation, and time modulation of the input signals.

(AP) 145-1993

**signal propagation delay or transmission delay** Total time for a signal to pass through the switching system. For digital-to-digital interfaces (DS1) where bit integrity is maintained, the delay is the time elapsed between transmission and reception of the bit that starts a known test pattern. For analog-to-analog, digital-to-analog, or analog-to-digital transmission, T1Q1 (a study group of the Exchange Carriers Standards Association) proposes that the delay be measured as the shift in time of the envelope of a 50% amplitude modulated test signal (or its digital equivalent) at the carrier frequency of minimum delay in the voiceband channel.

(COM) 973-1990w

**signal purity (network analyzers)** A measure of freedom from frequency components other than the desired measurement frequency. It includes harmonics, subharmonics, spurious mixer products, and unwanted components of signal or local oscillator leakage. *Note:* The resulting error in measurement is a function of the detection system and of the frequency response of the network under test, as well as the signal purity.

(IM) 378-1986w

**signal quality error heartbeat** A signal from the transceiver to a node peripheral indicating that the transceiver is functioning properly. *Note:* This term is contextually specific to IEEE Std 802.3.

(C) 610.7-1995

**signal queueing** When queueing is enabled for a signal, occurrences of that signal are queued in FIFO order and informa-

## master switch

**master switch (electric installations on shipboard)** A switch that dominates the operation of contactors, relays, or other remotely operated devices. *See also:* switch.

(IA) [60], 45-1983r

**master terminal (1)** A dedicated terminal that is reserved for the operator of the system or other authorized persons that are privileged to initiate conversations, and to control system-wide processes and operations. *Synonyms:* control terminal; operator console. (C) 610.10-1994

(2) *See also:* terminal, master. (PE/SUB) 999-1992

**master terminal unit (station control and data acquisition)** The master station of a supervisory control system. *See also:* master station; station.

(PE/SWG/SUB) C37.1-1994, C37.100-1992

**mast-type antenna for aircraft** A rigid antenna of streamlined cross section consisting essentially of a formed conductor or conductor and supporting body.

(EEC/PE) [119]

**mat (rotating machinery)** A randomly distributed unwoven felt of fibers in a sheetlike configuration having relatively uniform density and thickness. *See also:* rotor; stator.

(PE) [9]

**MAT** *See:* machine-aided translation.

**match (A)** A condition in which the values of corresponding components of two or more data items are equal. *See also:* hit. **(B)** To compare two or more data items to determine whether their corresponding components are equal as in definition "A." (C) 610.5-1990

**matched** A state applying to a sequence of zero or more characters when the characters in the sequence correspond to a sequence of characters defined by a BRE or ERE pattern.

(C/PA) 9945-2-1993

**matched condition** *See:* matched termination.

**matched filter** A filter that maximizes the output ratio of peak signal power to mean noise power. For white Gaussian noise, it has a frequency response that is the complex conjugate of the transmitted spectrum, or equivalently, has an impulse response that is the time inverse of the transmitted waveform.

(AE) 686-1990w

**matched generator insertion gain (waveguide)** A gain resulting from placing two ports of a network between a load having an arbitrary impedance and a matched generator. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched generator insertion loss.

(MTT) 146-1980w

**matched generator insertion loss (waveguide)** A loss resulting from placing two ports of a network between a load having an arbitrary impedance and a matched generator. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched generator insertion gain.

(MTT) 146-1980w

**matched impedances** Two impedances are matched when they are equal. *Note:* Two impedances associated with an electric network are matched when their resistance components are equal and when their reactance components are equal. *See also:* network analysis. (Std100) 270-1966w

**matched insertion gain (waveguide)** A gain resulting from placing two ports of a network between a matched generator and a matched load. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched insertion loss.

(MTT) 146-1980w

**matched insertion loss (waveguide)** A loss resulting from placing two ports of a network between a matched generator and a matched load. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched insertion gain.

(MTT) 146-1980w

## 632

## material dispersion parameter

**matched load insertion gain (waveguide)** A gain resulting from placing two ports of a network between a generator having an arbitrary impedance and a matched load. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched load insertion loss.

(MTT) 146-1980w

**matched load insertion loss (waveguide)** A loss resulting from placing two ports of a network between a generator having an arbitrary impedance and a matched load. It is the ratio of the power absorbed in the load when connected to the generator (reference power) to that when the network is inserted. *Contrast:* matched load insertion gain.

(MTT) 146-1980w

**matched terminated line (waveguide)** A transmission line having no reflected wave at any transverse section.

(MTT) 146-1980w

**matched termination (waveguide components)** A termination matched with regard to the impedance in a prescribed way; for example, a reflectionless termination or a conjugate termination.

(MTT) 147-1979w

**matched transmission line (data transmission)** A transmission line is said to be matched at any transverse section if there is no wave reflection at that section.

(PE) 599-1985w

**matched waveguide** *See:* matched terminated line.

**matching** *See:* image matching.

**matching, impedance** *See:* load matching.

**matching interaction** An instruction method employed by some computer-assisted instruction systems, in which the student is asked to match answers to questions.

(C) 610.2-1987

**matching, load** *See:* load matching.

**matching loss (1) (radar)** The loss in S/N (signal-to-noise) output relative to a matched filter, caused by using a filter of other than matched response to the transmitted signal. *Synonym:* mismatch loss. (AE) 686-1982s

**(2) (telecommunications)** The net probability of not being able to establish a network path between an originating line or incoming trunk and a terminating line or trunk when the terminating line or trunk is idle. *Synonym:* overflow loss.

(COM) 973-1990w

**matching section (waveguide) (waveguide transformer)** (transforming section) A length of waveguide of modified cross section, or with a metal or dielectric insert, used for impedance transformation. *See also:* waveguide.

(AP) [35]

**matching transformer (induction heater)** A transformer for matching the impedance of the load to the optimum output characteristic of the power source. (IA) 54-1955w

**material (nuclear power generating station)** A substance or combination of substances used as constituents in the manufacture of components, modules, or items. *Note:* This term applies specifically to the subject matter of IEEE Std 467-1980. (PE) 467-1980w

**material absorption** *See:* absorption.

**material dispersion (fiber optics)** That dispersion attributable to the wavelength dependence of the refractive index of material used to form the waveguide. Material dispersion is characterized by the material dispersion parameter  $M$ . *See also:* dispersion; distortion; material dispersion parameter; profile dispersion parameter; waveguide dispersion.

(Std100) 812-1984w

**material dispersion parameter (M) (fiber optics)**

$$M(\lambda) = -1/c(dN/d\lambda) = \lambda/c(d^2n/d\lambda^2)$$

where  $n$  is the refractive index,  $N$  is the group index:  $N = n - \lambda(dn/d\lambda)$ ,  $\lambda$  is the wavelength, and  $c$  is the velocity of light in vacuum. *Notes:* 1. For many optical waveguide materials,  $M$  is zero at a specific wavelength  $\lambda_0$ , usually found

## reconditioning of oil

882

## recording, instantaneous

(2) A general term covering the process of maintaining existing power switchgear equipment in operating condition as recommended by the manufacturer's instructions, using only the original manufacturer's recommended replacement parts, without altering the original design.

(PE/SWG) C37.100-1992

(3) The process of maintaining existing power switchgear equipment in operating condition as recommended by the manufacturer's instructions, using only the original manufacturers' designed parts. *Note*: Reverse engineered parts (designs copied from existing parts by other manufacturers) are not considered to be the original manufacturer's design or recommended replacement parts.

(PE/SWG) C37.59-1996

**reconditioning of oil** The mechanical removal of moisture and insoluble contaminants. (PE) C57.106-1991

**reconfiguration (1) (dual ring operation with wrapback reconfiguration)** A change of the path around which the token that is used for normal data transfer circulates.

(C/LM) 802.5c-1991

(2) **(DQDB subnetwork of a metropolitan area network)** The process by which the configuration control function activates and deactivates resources of a DQDB subnetwork to take account of a change in the operational status of a cluster, node, or transmission link in the subnetwork.

(C/LM) 8802-6-1994

(3) A strategy for repairing components in which failing components are switched out of operation and replaced by failure-free components. (C) 610.10-1994

**reconfiguration management** The management functions responsible for reconfiguration. This includes both dual ring management and any other management required for reconfiguration. (C/LM) 802.5c-1991

**reconstituted mica** *See*: mica paper.

**reconstruction (1)** Replacement of any portion of an existing installation by new equipment or construction. Does not include ordinary maintenance replacements.

(NESC) C2-1977s

(2) **(image processing and pattern recognition)** *See also*: image reconstruction. (C) 610.4-1990

**record (1) (data management) (software)** A set of data items, called fields, treated as a unit. For example, in stock control, the data for each invoice could constitute one record. *Synonym*: data record. *See also*: database record; entity.

(C) 610.12-1990, 610.5-1990

(2) The language-independent syntax for a family of datatypes constructed from a sequence of base datatypes, each associated with a name. A value of record datatype contains, for each name, a value of the corresponding base datatypes.

(C/PA) 1351-1994

(3) A set of related data items treated as a unit. For example, in stock control, the data for each invoice could constitute one record. (C) 610.7-1995

(4) To put data into a storage device. (C) 610.10-1994

(5) A datatype constructed from a sequence of base datatypes, each associated with a name. A record value contains, for each name, a value of the corresponding base datatype.

(C/PA) 1224.1-1993

(6) A collection of related data or words treated as a unit and saved in a position-dependent fashion within a file of other such units. (C/MM) 855-1990

**record condition (data management)** A conjunction of two or more item conditions such that the name of the data item in each condition is distinct. For example, "LASTNAME = 'JONES' and SEX = 'FEMALE.'" (C) 610.5-1990

**recorded announcement (telephone switching systems)** A prerecorded oral message received on a call.

(COM) 312-1977w

**recorded spot, X dimension (facsimile)** The effective recorded-spot dimension measured in the direction of the recorded line. *Notes*: 1. By effective dimension is meant the largest center-to-center spacing between recorded spots

which gives minimum peak-to-peak variation of density of the recorded line. 2. This term applies to that type of equipment which responds to a constant density in the subject copy by a succession of discrete recorded spots. *See also*: recording. (COM) 168-1956w

**recorded spot, Y dimension (facsimile)** The effective recorded-spot dimension measured perpendicularly to the recorded line. *Note*: By effective dimension is meant the largest center-to-center distance between recorded lines which gives minimum peak-to-peak variation of density across the recorded lines. *See also*: recording. (COM) 168-1956w

**recorded value** The value recorded by the marking device on the chart, with reference to the division lines marked on the chart. *See also*: accuracy rating. (EEC/PE) [119]

**recorder (1) (analog computer)** A device that makes a permanent record, usually graphic, of varying signals. *Synonym*: strip chart recorder. (C) 165-1977w, 610.10-1994

(2) **(facsimile)** That part of the facsimile receiver which performs the final conversion of electric picture signal to an image of the subject copy on the record medium. *See also*: facsimile; recording. (COM) 168-1956w

**recorder, strip-chart (analog computer) (hybrid computer linkage components)** A recorder in which one or more records are made simultaneously as a function of time. *See also*: electronic analog computer. (C) 165-1977w, 166-1977w

**recorder-warning tone (telephone switching systems)** A tone that indicates periodically that the conversion is being electrically recorded. (COM) 312-1977v

**record gap (1) (computers) (storage medium)** An area used to indicate the end of a record. (C) [20], [85]

(2) **(test, measurement, and diagnostic equipment)** An interval of space or time associated with a record to indicate or signal the end of the record. (MIL) [2]

(3) *See also*: interblock gap. (C) 610.5-1990

(4) *See also*: interblock gap. (C) 610.10-1994

**record head** *See*: read/write head.

**recording (1) (facsimile)** The process of converting the electrical signal to an image on the record medium. *See also*: electrochemical recording; electrolytic recording; electromechanical recording; electrostatic recording; electrothermal recording; magnetic recording; photosensitive recording.

(COM) 168-1956w

(2) The process of storing information on some storage medium for later retrieval. *See also*: magnetic recording; optical recording. (C) 610.10-1994

**recording area (1)** In micrographics, the maximum useful area of a microfilm or other medium that can record information, including the image as well as the document marks.

(C) 610.2-1987

(2) The area on a disk or storage medium on which information can be recorded. *Synonym*: recording zone. *Contrast*: handling zone. (C) 610.10-1994

**recording channel (electroacoustics)** The term refers to one of a number of independent recorders in a recording system or to independent recording tracks on a recording medium. *Note*: One or more channels may be used at the same time for covering different ranges of the transmitted frequency band, for multichannel recording, or for control purposes. *See also*: phonograph pickup. (SP) [32]

**recording-completing trunk (telephone switching systems)** A one-way trunk for operator recording, extending, and automatic completing of toll calls. (COM) 312-1977w

**recording demand meter** A demand meter that records on a chart the demand for each demand interval. *See also*: electricity meter. (EEC/PE) [119]

**recording density** The number of bits in a single linear track, measured in bits per unit of length or area of the recording medium. *Synonyms*: bit density; packing density; surface density. *See also*: track density. (C) 610.10-1994

**recording, instantaneous (mechanical recording)** A phonograph recording that is intended for direct reproduction with-

ENH & MATH SCI LIBRARY UGA



in such sound. *Note:* In practice, crosstalk may be measured either by the volume of the overheard sounds or by the magnitude of the coupling between the disturbed and the disturbing channels. In the latter case, to specify the volume of the overheard sounds, the volume in the disturbing channel must also be given. (PE) 599-1985w

(3) Unwanted electric signals injected into a circuit by stray coupling. (ELM) C12.1-1981

(4) (instrumentation and control equipment grounding in generating stations) The noise or extraneous signal caused by ac or dc pulse-type signals in adjacent circuits. (PE) 1050-1996

(5) (telecommunications) Undesired energy appearing in one path as a result of coupling from another path. Crosstalk is further classified as near-end, far-end, intelligible, or unintelligible. (COM) 1007-1991

(6) (telecommunications) Unwanted coupling between any two paths through the switching system. There are two situations of interest. In the first the crosstalk coupling loss of the system is linear; that is it is independent of applied disturbing signal level and is unaffected by other things, such as circuit noise. The other situation of interest arises with a digital switching system where the crosstalk coupling loss is nonlinear and is a function of applied disturbing signal level circuit noise, and encoder bias. Such systems will, under some conditions of circuit noise and coder bias, display crosstalk coupling loss that tends to decrease with decreasing disturbing signal level. *See also:* crosstalk coupling loss; equal-level crosstalk coupling loss. (COM) 973-1990w

(7) (multichannel) The ratio of the signal induced in one channel to a common signal applied to all other channels. (IM) 1057-1994

(8) The noise or extraneous signal caused by ac or pulse-type signals in adjacent circuits (measurement of power frequency magnetic fields). (PE/T&D) 1308-1994, 644-1994

(9) An electromagnetic field in the space surrounding a cable circuit created by an electrical signal. This field induces currents and electromotive forces in other circuits located close enough to the disturbing cable circuit to be affected. (PE) 1143-1994

(10) (A) A type of noise characterized by unwanted coupling of a signal or the interaction of signals on two adjacent channels. *See also:* near end crosstalk. (B) Undesired energy appearing in one signal path as a result of coupling from other signal paths. (C) 610.7-1995

(11) Crosstalk is undesired energy appearing in one signal path as a result of coupling from other signal paths. (C/LM) 8802-5-1995

**crosstalk coupling (crosstalk loss)** Cross coupling between speech communication channels or their component parts. *Note:* Crosstalk coupling is measured between specified points of the disturbing and disturbed circuits and is preferably expressed in decibels. *See also:* coupling. (EEC/PE) [119]

**crosstalk coupling loss (telecommunications)** The loss of the crosstalk path. *See also:* crosstalk. (COM) 973-1990w

**crosstalk, electron beam (charge-storage tubes)** Any spurious output signal that arises from scanning or from the input of information. *See also:* charge-storage tube. (ED) 158-1962w

**crosstalk loss** *See:* crosstalk coupling.

**crosstalk unit** Crosstalk coupling is sometimes expressed in crosstalk units through the relation

$$\text{Crosstalk units} = 10^{[6 - (L/20)]}$$

where  $L$  = crosstalk coupling in decibels. *Note:* For two circuits of equal impedance, the number of crosstalk units expresses the current in the disturbed circuit as millionths of the current in the disturbing circuit. *See also:* coupling. (EEC/PE) [119]

**crowbar** A protective circuit in a power distribution circuit that rapidly shorts the output voltage to ground when an over-voltage or other error condition occurs. (C) 610.10-1994

**CR-RC shaping (1) (charged-particle detectors) (germanium gamma-ray detectors) (x-ray energy spectrometers)** The pulse shaping present in an amplifier that has a simple high-pass filter consisting of a capacitor and a resistor together with a simple low-pass filter, separated by impedance isolation. (Pulse shaping in such an amplifier cuts off at 6 dB (decibels) per octave at both ends of the band.) (NPS) 301-1976s, 325-1986s, 759-1984r

(2) (semiconductor charged-particle detectors) In an amplifier, the pulse shaping produced by a single-section high-pass network followed by a single-section low-pass network, with the two networks separated by an isolating stage and with both networks having the same time constant. (NPS) 300-1988r

**CRS** *See:* configuration report server.

**CRT** *See:* cathode-ray tube.

**CRT display device** A display device that displays data onto a phosphor coated display screen using controlled electron beams within a CRT. *Note:* Raster display devices and random-scan display devices are two major categories of CRT display devices. *See also:* dark trace tube display device; penetration CRT display device; raster display device; storage tube display device. (C) 610.10-1994

**crude metal** Metal that contains impurities in sufficient quantities to make it unsuitable for specified purposes or that contains more valuable metals in sufficient quantities to justify their recovery. *See also:* electrorefining. (EEC/PE) [119]

**crust** A layer of solidified electrolyte. *See also:* fused electrolyte. (EEC/PE) [119]

**CRV** *See:* Code Rule Violation.

**cryogenics (1) (general)** The study and use of devices utilizing properties of materials near absolute-zero temperature. (C) [20], [85]

(2) (laser maser) The branch of physics dealing with very low temperatures. (LEO) 586-1980w

(3) A branch of technology concerned with devices that make use of the properties assumed by materials at temperatures near absolute zero. (C) 610.2-1987

**cryogenic storage** A type of storage that uses the superconductive and magnetic properties of certain materials at temperatures near absolute zero. (C) 610.10-1994

**cryotron (1)** A superconductive device in which current in one or more input circuits magnetically controls the superconducting-to-normal transition in one or more output circuits, provided the current in each output circuit is less than its critical value. *See also:* superconductivity. (ED) [46]

(2) A device that makes uses of the effects of extremely low temperatures on conductive materials such that small magnetic field changes can control large current changes. (C) 610.10-1994

**cryptographic checkvalue** Information that is derived by performing a cryptographic transformation on the data unit. *See also:* cryptography. (C/LM) 802.10-1992

**cryptography** The discipline embodying principles, means, and methods for the transformation of data in order to hide its information content, prevent its undetected modification, and/or prevent its unauthorized use. (C/LM) 802.10-1992

**crystal (A) (communication practice)** A piezoelectric crystal. (B) (communication practice) A piezoelectric crystal plate. (C) (communication practice) A crystal rectifier. (EEC/PE) [119]

**crystal-controlled oscillator** *See:* crystal oscillator.

**crystal diode** A rectifying element comprising a semiconducting crystal having two terminals designed for use in circuits in a manner analogous to that of electron-tube diodes. *See also:* rectifier. (EEC/PE) [119]

**crystal loudspeaker (piezoelectric loudspeaker)** A loudspeaker in which the mechanical displacements are produced by piezoelectric action. (EEC/PE) [119]

**crystal microphone (piezoelectric microphone)** A microphone that depends for its operation of the generation of an